A Model of Search with Price Discrimination*

Natalia Fabra
Universidad Carlos III and CEPR

Mar Reguant
Northwestern, CEPR and NBER

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Abstract

We introduce heterogeneity in buyers' size into a model of simultaneous search. Buyers' differences in their willigness to search give rise to price discrimination, even if their valuations for the good are equal. We shed light on three related questions: (i) what is the relationship between prices and buyers' size? (ii) what are the effects of reducing search costs?, and (iii) who benefits and who is hurt by price discrimination? The answers critically depend on the elasticity of the search cost distribution. Interestingly, for normally distributed search costs, (i) there is an inverted U-shape relationship between prices and buyers' size, (ii) when search costs go down, the prices charged to small buyers do not fall as much as those charged to the large ones (and can even go up), and (iii) price discrimination benefits small and large buyers, at the expense of the medium-size buyers.

Keywords: price discrimination, search, bid solicitation.

1 Introduction

This paper studies the interaction between price discrimination and search costs in markets characterized by imperfect competition. While both search and price discrimination have long been studied as distinct issues, less is known about how they interact. Online markets provide an example in which such interaction becomes particularly meaningful. Search costs and the ability to price discriminate are key determinants of prices in these markets, an issue that has become a major focus of recent competition policy cases.[1] The internet provides a vast amount of consumer data

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[1]There is a long list of recent competition policy decisions regarding online markets, both in Europe as well as in the US. In the EU, in May 2015 the European Commission launched an antitrust investigation into e-commerce; its preliminary findings were published in September 2016 (European Commission, 2016). Also, in July 2016, the European Commission sent an Statement of Objections to Google concluding that Google had abused its dominant position by favouring its comparison shopping service in its search result pages. Competitive concerns over e-commerce have also arisen in the US. For instance, in April 2015 an e-commerce seller of posters was found guilty for price fixing through Amazon Marketplace.

which firms can use to tailor their pricing policies, and the internet allows firms to limit consumers' arbitrage opportunities, e.g. through the use of practices such as geo-blocking,[2] among others. And, even though the internet has drastically reduced search frictions, there is ample evidence indicating that a large amount of online consumers do not engage in active search.[3]

In this paper we build an oligopoly model in which heterogenous buyers engage in costly search. In particular, we assume that buyers differ in their sizes (which are observable)[4] as well as in their search costs (which are non-observable). In order to stress that search frictions can be a source of price discrimination, we assume away other incentives to price discriminate. In particular, we assume that all buyers - regardless of their size - have the same *willingness to pay*, i.e., in the absence of search frictions, they would all face equal prices. Instead, differences in buyers' size introduce ex-ante differences in their *willingness to search*, which in turn affect the intensity of rivalry among sellers when competing for either large or small buyers.

A buyer's *willingness to search* depends on her size and on her search cost: a buyer's size provides a signal about her gains from search, but her decision to engage in search conveys countervailing information about her search costs. On the one hand, large buyers stand to gain more from search since the potential price savings apply to a bigger demand ("gain-from-search effect"). However, precisely because they gain more from search, the large buyers' decision to participate in search is less informative about their search costs ("signalling-through-participation effect"). Which of these two countervailing effects dominates determines whether sellers compete more or less aggressively to serve larger buyers- a key issue when addressing the three main questions of this paper.

First, we want to understand the relationship between prices, search intensity and consumers' sizes. We show that this issue critically depends on the shape of the search cost distribution. More specifically, the relative importance of the "gains-from-search effect" versus the "signaling-through-participation effect" depends on the elasticity of the search cost distribution. When this elasticity is decreasing (increasing), bigger (smaller) buyers are expected to search more, which in turn leads sellers to compete more aggressively to serve them. Intuitively, when the search cost distribution is more elastic at the lower (upper) deciles of the search cost distribution, an increase in size affects more the buyers' search intensity ("gains-from-search effect") than the likelihood of search ("signaling-through-participation effect"). In turn, this implies that sellers compete more aggressively to serve bigger (smaller) buyers. For iso-elastic functions (e.g. when search costs are uniformly distributed), the two effects cancel out leading to no price discrimination in equilibrium.[5]

Our analysis thus suggests that the elasticity of the search cost distribution plays a key role in determining whether large buyers possess buyer power.[6] Several commonly-used distributions,

[2]The European Commission (2016) documents that geo-blocking is widely used in e-commerce across the EU.

[3]For instance, in the market for online books, De los Santos et al. (2012) report that 74% of all consumers buy at the first store they visit without searching any further.

[4]Similar results would arise if we introduced other sort of observable differences across buyers: e.g., differences in valuations, or differences in the search cost distribution functions.

[5]This discussion refers to the ranking of prices conditional on search, not to buyers' expected utility prior to search. Since large buyers are relatively more likely to engage in search, their expected utilities tend to be higher even when they do not possess buyer power.

[6]To be sure, estimating the distribution is not straightforward. See Hortaçsu and Syverson (2004), Hong and

such as the exponential or the Pareto distribution, depict monotonically decreasing elasticities, thus implying that larger buyers indeed pay lower prices. However, other distributions have non-monotonic elasticities (e.g. the Normal or the Gamma distributions), thus leading to prices that are not always decreasing in size.[7] Interestingly, with normally distributed search costs, the small and the large consumers pay the lowest prices. The medium-sized consumers are not small enough to benefit from signaling a low search cost when they engage in search, and they are not large enough to benefit from high gains from search. In sum, our results support the hypothesis that larger buyers may have buyer power, but only under certain circumstances. In line with existing evidence (Sorensen, 2000; Ellison and Snyder, 2010; Grennan, 2013; and the UK Competition Commission 2008's inquiry into the groceries market), this suggests that the relationship between buyers' size and prices must be assessed on an industry-by-industry basis. Our results complement those of the bilateral contracting literature, without relying on the distribution of bargaining power between buyers and sellers, nor on the impact of buyer's size on the parties' outside option (Katz, 1987; and Inderst and Montez, 2016).[8]

Second, we want to understand the effects of reduced search costs on the prices paid by the various consumers. This issue has become particularly important as the widespread of online retailing has reduced search cost for a wide variety of products and services (Baye et al., 2004). Our analysis reveals that the price impact of a fall in search costs need not be uniform across buyers. When cost shifts are additive (i.e., the search cost density moves in a parallel fashion), a reduction in search costs typically depresses prices for all consumers, but not in the same magnitude. In contrast, when cost shifts are multiplicative, the effect of reduced search costs is analogous to the effect of increased size, bringing us back to the question of whether search cost elasticities are decreasing or increasing over the relevant range. Unlike conventional wisdom, a reduction in search costs may lead small consumers to pay higher prices when the search cost distribution depicts increasing elasticities. The intuition goes in line with our previous results: when search costs go down, the participation decisions of the small consumers become less informative about their search costs being low. This weakens the "signaling-through-participation effect" and leads to higher prices. For large buyers, the weakening of the "signaling-through-participation effect" is less pronounced because their participation decisions are less sensitive to the reduction in search costs.

Last, in order to shed light on the distributional effects of price discrimination, the paper compares equilibrium market outcomes under uniform prices versus price discrimination. This

Shum (2006), Moraga and Wildenbeest (2008) and De los Santos et al. (2012) for papers that perform this task in various contexts.

[7] For instance, in the case of online books, Hong and Shum (2006) and De los Santos et al. (2012) find that the distribution of search costs depicts non-monotonic elasticities.

[8] In the former, buyers' outside option is to procure the good from an alternative supplier. Large buyers' stand at a stronger bargaining position since accessing the alternative supplier entails a fixed cost. In the latter, the buyer's outside option is to relocate demand across various sellers. This creates countervailing effects as large buyers find it difficult to relocate all their demand to alternative suppliers, while sellers find it difficult to fill their capacity if the negotiations with large buyers do not go through. These forces imply that size provides a competitive advantage for buyers only when their bargaining power is sufficiently high.

issue has become particularly relevant with the widespread of internet. Whether online retailers should be banned from price discriminating (e.g. by forcing them to relax the constraints on consumers' arbitrage, such as geo-blocking), or whether they should be banned from using most-favoured-nation clauses (MFNs) that lead to price uniformity, are contentious issues in the policy arena.[9] Beyond their welfare implications, our analysis reveals that the effects are likely to differ across consumer types. While it is well known that price discrimination has distributive effects, the novelty of our analysis is to point out that a ban on price discrimination does not necessarily benefit the small buyers. Indeed, their prices are reduced when price discrimination is banned, but only when the search cost distribution depicts decreasing elasticities. Instead, if search costs are normally distributed, a ban on price discrimination would reduce the prices for the medium-sized consumers at the expense of the higher prices paid by the small and large ones.

Related literature Since the seminal work of Diamond (1971), a broad literature has emerged analyzing the impact of consumers' search costs on market outcomes. However, to the best of our knowledge, all existing papers in the search literature assume *ex-ante* identical consumers, i.e., they all have unit demands and equal valuations of the good, and either their search costs are also equal, or they are unobserved by the sellers. Hence, these papers are not well equipped to analyze the interaction between search costs and price discrimination. Together with Fabra and Montero (2017),[10] we are the first in explicitly allowing for price discrimination in a model of search.[11] It is well known that search costs generate equilibrium price dispersion (including Varian, 1980; Burdett and Judd, 1983; Stahl, 1989; Janssen and Moraga, 2004, among others), which in turn implies that consumers who search more end up paying lower prices even when charged equal prices. However, such price differences due to price dispersion are not to be confounded with the price differences that arise due to price discrimination *per se*, i.e., when sellers charge different prices to different buyers.

Another aspect further distinguishes our paper from most of the existing search literature: instead of taking search as given, we allow buyers to endogenously choose whether to engage in search or not. Endogenous participation is a key driver of price discrimination because the decision to participate in search conveys different information regarding search costs depending on the buyer's size.[12] A notable exception is Moraga et al. (2017), who develop a sequential search model with differentiated products and endogenous search. They show that a reduction in search

[9] Fort instance, in January 2015, the UK Competition and Markets Authority opened a call for information on the commercial use of consumer data. Furthermore, they have discouraged the use of wide MFNs or price parity provisions in areas such as motor insurance. As already mentioned, the European Commission has also raised concerns about residence-based price discrimination in several occasions.

[10] This paper deals with third degree price discrimination where as Fabra and Montero (2017) allows for second degree price discrimination through quality choices.

[11] In a model of sequential search, Janssen and Rushidi (2018) analyze price discrimination by wholesale firms (yet, at the downstream market, firms charge equal prices to all customers). Charging different prices to the various retailers creates price dispersion in the downstream market, which in turn induces more search and stronger retail market competition, to the benefit of wholesalers.

[12] This driver is absent in the majority of the search models as these assume that buyers search at least once (either because search costs are sufficiently low, or because the first quote is for free).

costs can give rise to price increases as changes in the pool of consumers who engage in search make the market demand more inelastic. Our analysis shares some of the driving forces in Moraga et al. (2017), but relies on different assumptions and pursues different objectives. In particular, in Moraga et al. (2017) firms have no ex-ante information about individual buyers' characteristics, leaving no scope for price discrimination. And whereas we both analyze the issue of how market prices respond to changes in search costs, our focus is on how such effects differ across buyer types.

The recent literature on bidder solicitation also endogenizes search decisions. In particular, Lauermann and Wolinsky (2017) allows buyers to choose how many prices to solicit from a set of potential sellers at a *solicitation cost*, similar to the *search cost* in our paper (see also Lauermann and Wolinsky, 2016). The two analyses differ in one key aspect: in Lauermann and Wolinsky (2017), the buyer is privately informed about the value of the good (common-value setting); in our paper, she is privately informed about the solicitation or search cost (private-value setting). Despite these differences, both setups share an important ingredient: the buyer's participation decision (in solicitation or in search) provides a signal about the buyer's private information. In Lauermann and Wolinsky (2017), sellers are more likely to be solicited when the value of the good is low; hence, the solicitation effect softens competition. In our model, the signaling-through-participation effect enhances competition more for small than for large buyers as the decision to engage in search by a small buyer is more informative about her search costs being low.

Last, since we explore the relationship between prices and buyers' size, our paper is also related to the literature on buyer power. This literature typically explores the relationship between prices and buyers' size through models of bilateral negotiations, absent search frictions. Accordingly, one of the novelties of the current paper is to approach the same question through the lens of a search model. With only one exception - which we discuss below -, this literature unambiguously concludes that larger buyers obtain lower per unit prices (see Snyder, 2002 for a survey). Larger buyers have more bargaining power because of scale economies, because the marginal surplus from transactions involving large quantities is higher. We identify sufficient conditions on the magnitude and distribution of search costs under which larger buyers obtain lower prices under assumptions for which the existing literature would predict no buyer power: namely, constant returns to scale, take-it-or-leave it offers by sellers, and symmetric outside options for all buyers and sellers (Horn and Wolinsky, 1986, Stole and Zwiebel, 1996, Chipty and Snyder, 1999, Inderst and Wey, 2003, Raskovich, 2003, Sheffman and Spiller, 1992, and Inderst and Wey, 2007, among others). However, we also identify properties of the search cost distribution for which smaller buyers pay lower prices as compared to the large ones. This points at the importance of understanding consumers' search to assess whether large buyers posses buyer power.

The papers cited above assume a single seller, and hence are not well suited to capture the idea that buyer power stems from increased rivalry among sellers. Only few papers allow for multiple sellers. In Snyder (1999), buyer power reduces prices because sellers find it more difficult to collude when serving larger buyers. In Dana (2012) and Inderst and Shaffer (2007), buyer groups obtain lower prices than stand-alone buyers because, by pooling buyers with different preferences, the intensity of the group's preference over a specific product is reduced. In turn, this enhances the group's bargaining power and results in lower prices.

Inderst and Montez (2016) is the only paper we are aware of that, like ours, concludes that size is not a univocal source of buyer power.[13] They study bilateral bargaining between various buyers and sellers among which there is *mutual dependency*: in case of a failure in their negotiations, large buyers find it difficult to reallocate their demand to alternative sellers, while sellers find it difficult to reallocate their sales to alternative buyers. Thus, buyer power benefits the side which is least dependent on the other. In particular, Inderst and Montez (2016) show that larger buyers face higher per unit prices when sellers have sufficiently high bargaining power and their marginal costs are increasing (including the case of stringent capacity constraints). Our paper provides a similar conclusion as theirs - prices need not be decreasing in size - but the drivers are quite different. In particular, in our model, buyers have no bargaining power irrespective of their size (sellers make take-it-or-leave it offers), marginal costs are constant and sellers do not face capacity constraints. Yet, our model predicts that larger buyers can end up paying higher prices (conditional on search) whenever the participation decisions of the smaller buyers signal sufficiently low search costs. We also share with Inderst and Montez (2016) their main policy conclusion: the assessment of buyer power should be industry-specific. Their model suggests that the shape of the marginal cost function, the distribution of bargaining power and the degree of flexibility matter. To these, we add the magnitude of search costs (relative to buyers' size) and the distribution of such costs as relevant determinants of buyer power as well.

The remainder of the paper is structured as follows. In Section 2, we construct and solve the model; we first characterize sellers' pricing decisions (section 2.1) and buyers' search behavior (section 2.2), and then prove existence of the equilibrium (section 2.3). Comparative statics of equilibrium prices with respect to buyers' size and with respect to the search cost distribution are performed in sections 3 and 4 respectively. In section 5, we characterize the equilibrium when price discrimination is banned, and compare the results to the case when price discrimination is allowed. Section 6 of the paper concludes. All proofs are included in the appendix.

2 Model description

Consider a buyer who is willing to buy q units of a homogeneous good in a market with two competing sellers.[14] Her per-unit valuation of consuming the good is normalized to 1. The buyer can decide whether to *search* to find out the prices of the competing sellers, or not.[15] If she does

[13] There is also a literature analyzing whether the discounts obtained by larger buyers are passed on to final consumers (Dobson and Waterson (1997), von Ungern-Sternberg (1996)). These papers show that increased concentration in the downstream market implies lower wholesale prices due to buyer power, but this does not always benefit consumers as increased concentration downstream also means weakened competition. We abstract from this issue as the buyers in our model are the final consumers.

[14] The results of the model extend to the case of $N > 2$ sellers.

[15] Alternatively, one could assume that a buyer who does not search can buy the good at a default price normalized to 1. This would apply to several settings, e.g. in energy markets, consumers typically have access to energy at a

not search, her reservation utility is normalized to zero. The marginal cost of producing the good is also normalized to zero.[16]

The buyer incurs a search cost c to observe the price of each seller.[17] Her search cost is private information, but it is common knowledge that c is drawn from the cumulative distribution function $G(c)$, with density $g(c) > 0$, in the interval $[\underline{c}, \overline{c})$. We assume $\underline{c} \leq 0$ and $\overline{c} \geq q$ to focus on the interesting case in which some buyer types engage in search with probability one, while others never engage in search.[18]

The timing of the game is as follows. First, the buyer observes her realized search cost and takes her search decisions: not to search, search once (picking one of the two sellers at random), or search twice. Sellers observe the size of the buyer q but do not observe her search cost nor her search behavior. Second, each seller chooses a price b and the buyer buys from the seller that offered the lowest price among the ones she observes (ties are broken randomly).[19]

We examine (symmetric) equilibria in which the buyer maximizes her utility given her correct beliefs about the sellers' pricing behavior, and the sellers maximize their profits given their correct beliefs about the buyer's search behavior.

2.1 Pricing decisions

Let us first characterize sellers' pricing behavior given their expectations about the buyer's search behavior. Consider a seller's pricing decision. The seller does not observe the buyer's search behavior but believes that, *conditionally on search*, the buyer has searched once with probability ρ. In equilibrium, this expectation must turn out to be correct. To the extent that the buyer's size conveys different information about the her search behavior, sellers might hold different beliefs about ρ depending on q. In this section, we take ρ as a parameter, and we will endogenize it later in section 2.3.

Our first result shows that in equilibrium, conditional on search, some buyer types search once while others search twice. Hence, each seller does not know with certainty whether the buyer will not observe his price, whether he will be a monopolist over the buyer, or whether he will be competing with the other seller. The only information that sellers can infer is that the buyer's search cost must be low enough for her to be willing to search at least once, given her size.

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regulated price, and consumers also have the option to search for competing retailers. The results of the model would remain the same.

[16]Both normalizations are *w.l.o.g* as long as valuations and constant marginal are assumed constant. We assume they are both constant so as not to bias the relationship between buyers' size and equilibrium outcomes through the assumptions on the shape of the utility or cost functions.

[17]We could introduce scale economies (or diseconomies) in search by assuming that the second quote costs δc, with $\delta < 1$ ($\delta > 1$). Our main results would remain qualitatively unchanged.

[18]Unlike other models in the search literature (e.g. Varian's model of sales), shoppers are not essential in our model. Indeed, our model has $\underline{c} = 0$ (i.e., there are no shoppers) as a particular case. However, the mass of shoppers may have an impact on the nature of the equilibria in interesting dimensions (see section 3).

[19]Since sellers can condition their price offers to the size of the buyer, we are implicitly assuming that sellers engage in third-degree price discrimination and buyers cannot mis-report their size.

Lemma 1 *In a SPNE, if the buyer engages in search, the probability that she searches once is* $\rho \in (0,1)$. *As a consequence, there does not exist pure-strategy equilibria in prices.*

The intuition for this result is well known (Burdett and Judd, 1983). On the one hand, if the buyer searches once with certainty ($\rho = 1$), the seller charges the monopoly price. However, as this would leave no surplus for the buyer, she would not search in the first place (Diamond's paradox). On the other hand, if the buyer searches twice with certainty ($\rho = 0$), sellers engage in Bertrand competition. Since all sellers would then price at marginal costs, the buyer would have incentives to search only once. As a consequence, neither the monopoly price nor the competitive price can be sustained in a SPNE.[20,21]

More generally, $\rho \in (0,1)$ rules out the existence of pure strategy equilibria regardless of the buyer's size:[22] starting from any arbitrary price pair, sellers would like to undercut each other until prices are so low that it becomes optimal for a seller to price at the consumer's maximum willingness to pay for the good. However, if one seller is pricing at that level, it becomes profitable for the other seller to slightly undercut it. Therefore, the equilibrium has to be in mixed strategies and, using the same Bertrand argument, it must be atomless.[23]

To distinguish the price offered by the seller from the price actually chosen by the buyer, we refer the former as a *quote*. Let sellers use the (symmetric) quote distribution $F(b)$. Conditional on the buyer's decision to search, a seller's expected profits form pricing at b are given by

$$\pi(b) = bq\left[\rho/2 + (1-\rho)(1-F(b))\right].$$

A seller's mixed strategy strikes a balance between two opposing forces. On the one hand, sellers benefit from charging a high price to a buyer who has only searched once. This event occurs with probability $\rho/2$ (recall that both sellers are equally likely to be picked by the buyer).[24] On the other hand, sellers also benefit from charging a low price, as it is thus more likely to be chosen by a buyer who has searched twice. This event occurs with probability $(1-\rho)(1-F(b))$. Proposition 1 below characterizes the equilibrium quote distribution.[25]

Proposition 1 *Assume* $\rho \in (0,1)$. *There is a unique symmetric equilibrium quote distribution. It*

[20] Note that this result would remain unchanged in the case of $N > 2$ firms.

[21] If the first quote is free (as in Burdett and Judd, 1983), there always exists an equilibrium where all firms charge the monopoly price.

[22] This result is reminiscent of Janssen and Rasmusen (2002) in which with an exogenously given probability, rival firms may be inactive. In contrast, in this paper we endogenize this probability through the analysis of consumers' search (see next section).

[23] This is in contrast to Lauermann and Wolinsky (2016), in which the common value assumption gives rise to at atom in bidders' strategies. This atom implies a failure of competition to aggregate information even when search costs are very low and competition is strong.

[24] Note that profits are represented conditional on a buyer searching. Results are unaffected if we instead compute expected profits conditional on the firm having received a quote request, i.e., if we re-scale profits by $1/(1-\rho/2)$.

[25] Hong and Shum (2006) and Moraga and Wildenbeest (2008) derive the same distribution for the case with $N > 2$ firms, but treat the ρ parameters as exogenously given. Also, they only consider consumers with unit demands.

is atomless, and it is given by

$$F(b) = 1 - \frac{1}{2}\frac{\rho}{1-\rho}\frac{1-b}{b}$$

with compact support $b \in \left[\frac{\rho}{2-\rho}, 1\right]$.

Interestingly, the buyer's size does not enter directly into the sellers' pricing strategy. However, this does not imply that sellers choose the same prices regardless of the buyer's size. As we will see in the next section, the effect of q on price quotes is channeled through ρ, i.e., the sellers' expectation of the buyer's search strategy, which depends on the buyer's size. Indeed, an increase in ρ shifts the whole quote distribution to the right in a FOSD sense, also increasing the lower bound of the quote support. Intuitively, an increase in ρ implies that sellers price less aggressively, leading to higher expected quotes.[26] In other words, a buyer who is expected to search more observes lower quotes, regardless of her actual search intensity.

Clearly, sellers never quote prices above the buyer's maximum willingness to pay. Therefore, conditionally on search, the buyer always buys from one of the sellers. In particular, if a buyer observes only one quote (which conditionally on search, occurs with probability ρ), she simply pays that quote, which in expectation equals

$$E[b] = \int_{\underline{b}}^{1} b dF(b).$$

However, if she observes two quotes (with probability $1-\rho$), she pays the minimum of the two, which in expectation equals

$$E[\min\{b_1, b_2\}] = \int_{\underline{b}}^{1} 2b(1-F(b))\, dF(b).$$

Putting both pieces together and using the quote distribution characterized in Proposition 1, the next lemma computes the expected price paid by a buyer conditional on search.

Lemma 2 *Conditional on search, the expected price paid by the buyer is* ρ.

Even though sellers use mixed strategies without observing the buyer's search behavior, in expectation, prices are the same *as if* sellers observed it. In such a case, sellers would charge the monopoly price (equal to one) with probability ρ and would charge the Bertrand price (equal to zero) with probability $(1-\rho)$. However, and in contrast with the case with a known number of competitors, this model generates equilibrium price dispersion (as in Burdett and Judd, 1983).

Importantly, the Lemma above implies that expected prices conditional on search are increasing in ρ. The reason is two-fold. First, there is a simple *probability effect*: the higher ρ, the less likely it is that the buyer can compare the two quotes and choose the lowest one. And second, there is a

[26]The fact that ρ shrinks the quote support does not mean that dispersion is reduced because ρ also affects the shape of the distribution. Indeed, an increase in ρ has a non-monotonic effect on the dispersion of quotes: it first increases and it then decreases as ρ goes up.

competition effect: the higher ρ, the weaker is the competition between the sellers and hence the higher the expected quotes. Thus, expected search behavior has a direct translation on how total surplus is distributed: sellers make profits ρq, and the buyer obtains utility $(1-\rho)\,q$.[27]

So far, we have characterized equilibrium outcomes as a function of ρ. Interestingly, we have found that expected prices conditional on search do not depend directly on q. However, since ρ is to be endogenously determined once we incorporate the buyer's optimal search behavior, q will indirectly affect prices both through its effect on ρ, as well as possibly through its effect on the buyer's decision to search. To analyze this in detail, we now turn our attention to characterizing buyers' optimal search behavior.

2.2 Search decisions

Once the buyer has observed her realized search cost c, she has to decide whether to engage in search and if so, whether to search once or twice. Her utility from searching once or twice is respectively given by u_1 and u_2 (recall that the buyer's valuation for the good is normalized to 1),

$$\begin{aligned} u_1 &= (1 - E[b])\,q - c \\ u_2 &= (1 - E[\min\{b_1, b_2\}])\,q - 2c. \end{aligned}$$

The next Proposition characterizes the buyer's optimal search strategy.

Proposition 2 *The buyer's optimal search strategy follows a cut-off rule: for given $\rho \in (0,1)$, there exist $0 < c_1 < c_0 < q$ such that the buyer does not search if $c > c_0$, she searches once if $c \in (c_1, c_0]$ or she searches twice otherwise.*

For the buyer to find it optimal to search, her utility from doing so must be non-negative, i.e., the expected gross utility from search must exceed her search cost. This amounts to

$$c \le c_0 = (1 - E[b])\,q.$$

Using the distribution of price quotes characterized in Proposition 1, c_0 can be expressed as

$$c_0 = \left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\frac{2-\rho}{\rho}\right) q.$$

Holding ρ constant, c_0 is increasing in q: *ceteris paribus*, larger buyers gain more from participating in search. Also, the threshold c_0 is decreasing in ρ: a higher ρ reflects higher expected prices, making search less attractive. On one extreme, if $\rho = 0$, expected prices are equal to (zero) marginal costs;

[27] Search intensity also affects price dispersion. Indeed, the coefficient of variation (measured as the ratio between the standard deviation and the mean of prices, see Sorensen (2000), among others) is monotonically decreasing in ρ. Hence, one should expected lower price dispersion in markets with higher prices –a result that is reminiscent of Stigler's seminal contribution despite the fact that he assumed the quote distribution to be exogenously given, i.e., independent of the intensity of search.

hence, the buyer engages in search whenever $c \leq c_0 = q$. On the other extreme, if $\rho = 1$, expected prices are equal to the buyer's valuation; hence, the buyer engages in search only if it is free, $c \leq c_0 = 0$. In equilibrium, since $\rho \in (0,1)$ (Lemma 1), $c_0 \in (0, q)$.

Consider now the decision to search twice: for it to be optimal, the buyer's search cost has to be below the expected savings from observing two rather than just one quote. This amounts to

$$c \leq c_1 = \left(E\left[b\right] - E\left[\min\left\{b_1, b_2\right\}\right]\right) q.$$

Using Proposition 1, c_1 can be expressed as

$$c_1 = \frac{\rho}{1-\rho}\left(\frac{1}{2}\frac{1}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right) - 1\right) q.$$

For given ρ, the threshold c_1 is also increasing in q because the gains from search are proportional to q. However, unlike c_0, the threshold c_1 is non-monotonic in ρ given that ρ affects expected prices both when the buyer searches once as well as when she searches twice, with both effects pushing c_1 in opposite directions.[28]

2.3 Equilibrium characterization

In equilibrium, the buyer's beliefs about the distribution of quotes in the market must be consistent with sellers' actual pricing behavior. Likewise, conditional on search, sellers' beliefs about the buyer's search strategy must be consistent with her actual search behavior. Thus, in equilibrium, the following condition must satisfied:

$$\rho^* = 1 - G\left(c_1\left(q, \rho^*\right) \middle| \, c \leq c_0\left(q, \rho^*\right)\right),$$

where we have explicitly added the arguments (q, ρ) to stress that the search thresholds c_0 and c_1 depend on the buyer's size as well as on her expected search behavior.

Importantly, since the buyer only finds it optimal to search when her search cost realization is sufficiently low, her participation decision signals that his search cost is below c_0. The equilibrium condition incorporates this, as the distribution that sellers use to compute the buyer's expected search behavior is truncated at c_0.[29] More succinctly, the above expression can be re-written as

$$\rho^* = 1 - \frac{G\left(c_1^*\right)}{G\left(c_0^*\right)} \in (0,1). \tag{1}$$

Together with our previous results, the solution to equation (1) completes the characterization of the SPNE. The following Proposition guarantees that there always exists a solution to equation

[28] In particular, c_1 first increases and then decreases in ρ. Furthermore, $\lim_{\rho \to 0} c_1 = \lim_{\rho \to 1} c_1 = 0$. See the appendix for details.

[29] If the first quote is for free, as assumed in various papers of the search literature, participation is not an issue. In this case, the equilibrium condition is simply $\rho^* = 1 - G\left(c_1^*\right)$. It is easy to see that in this case a unique equilibrium always exists, with ρ^* always decreasing in q.

(1), which is interior (in line with Lemma 1).

Proposition 3 *There exists a symmetric SPNE in which sellers price as stated in Proposition 1 and buyers search as stated in Proposition 2. Conditional on search, the probability that the buyer searches once is given by the solution to* (1).

The proof of the Proposition also identifies sufficient conditions for this solution to be unique. In particular, the equilibrium is guaranteed to be unique for all log-convex G; or if G is log-concave, when it is not too concave.[30] The family of distribution functions for which the solution is unique is very broad, including the uniform, normal, and exponential distributions, among others. In the rest of the paper, we assume that the search cost distribution is such that the equilibrium is unique.

3 The effects of buyers' size

To understand the interaction between prices and buyers' size, we perform a series of comparative statics. We start by focusing on the expected price conditional on search, ρ^* (Lemma 2). For this purpose, it is useful to define the elasticity of the search cost distribution G,[31]

$$\varepsilon(c) \equiv c\frac{g(c)}{G(c)}.$$

Proposition 4 *In equilibrium, the expected price conditional on search, ρ^*, is (i) decreasing in q if $\varepsilon(c_1^*) > \varepsilon(c_0^*)$; (ii) increasing in q if $\varepsilon(c_1^*) < \varepsilon(c_0^*)$; and (iii) constant in q if $\varepsilon(c_1^*) = \varepsilon(c_0^*)$.*

Sellers compete more aggressively for buyers who are expected to search more. But, is it always the case that sellers expect larger buyers to search more? The answer is no. To understand why, let us decompose a buyer's willingness to search in two components: the *gains from search* and the *costs of search*. On the one hand, the gains from search are known to be higher for larger buyers given that any potential price savings achieved through search are proportional to their size q. We refer to this as the "gains-from-search effect". On the other hand, even if all buyers have ex-ante equal expected search costs, their decisions to engage in search convey different information regarding their search costs. Precisely because small buyers stand to gain less from search, sellers expect those small buyers who engage in search to have relatively lower search costs. We refer to this as the "signalling-through-participation effect". Thus, whether sellers expect larger or smaller buyers to search more, critically depends on the interplay between these two countervailing effects.[32]

[30]As shown in the Appendix, this guarantees that the slope of the schedule $1 - G(c_1)/G(c_0)$ is either negative or, if positively sloped, its slope is never above 1.

[31]Note that the elasticity can also be expressed as $\varepsilon(c) \equiv cr(c)$, where $r = g/G$ is the reverse hazard rate. The elasticity is decreasing in c if $\partial r/\partial c = -r/c$, i.e., the reverse hazard rate has to be sufficiently decreasing in c. For G log-convex, the elasticity is everywhere increasing.

[32]If the buyer can observe the first quote for free, the equilibrium comparative statics show that $\partial\rho^*/\partial q = -(\partial c_1^*/\partial q)g(c_1^*) < 0$, i.e., larger buyers always search more and always pay lower prices. Thus, the endogeneity of search decisions is crucial for the comparative statics reported in Proposition 4.

The "gains-from-search" and the "signalling-through-participation" effects work in opposite directions. Whether one effect or the other dominates, depends on the shape of G. In particular, it depends on the elasticity of the search cost distribution.[33] Suppose that the elasticity of $G(c)$ at c_1^* is greater (lower) than at c_0^*. Hence, from the sellers' point of view, an increase in q increases the probability that the buyer asks for two quotes, $G(c_1^*)$, more (less) than it increases the probability that the buyer engages in search, $G(c_0^*)$. Hence, the conditional probability that the buyer has searched only once, ρ^*, decreases (increases) in q. In sum, if the elasticity of the search cost distribution is higher (lower) at c_1^* than at c_0^*, sellers expect large buyers to search more (less) intensively and hence compete more (less) aggressively to serve them. This implies that larger buyers pay lower (higher) expected prices as compared to the smaller buyers.

If the elasticity is the same at these two thresholds, the "gains-from-search" and the "signalling-through-participation" effects cancel out. Hence, sellers expect all buyers to search with the same intensity, regardless of their size.[34]

The shape of the search cost distribution ultimately determines whether $\varepsilon(c_1^*)$ is higher or smaller than $\varepsilon(c_0^*)$, and thus the comparative statics of expected prices conditional on search with respect to q. A sufficient condition for prices to be decreasing, increasing or constant in q is that the elasticity of $G(c)$ is either decreasing, increasing or constant, respectively. There are several distribution functions with monotone elasticities. For instance, under the exponential or the Pareto distribution, elasticities decrease monotonically in c, implying that larger consumers pay lower prices. In contrast, if search costs are uniformly distributed with $\underline{c} < 0$ (i.e., some buyers enjoy searching), the elasticity is increasing in c in the interior range (for $c > 0$), so that larger consumers pay higher prices. However, in the absence of shoppers, i.e., $\underline{c} = 0$, the elasticity is constant so that all consumers pay the same price.

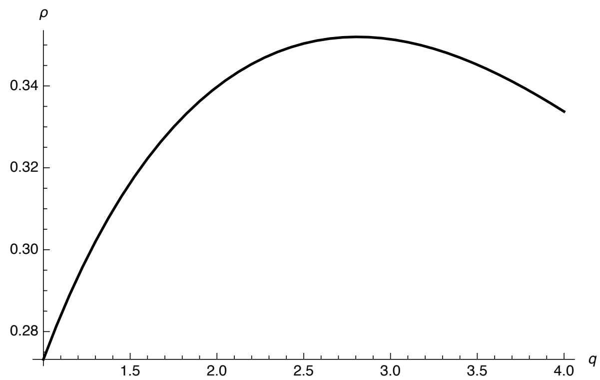
For many commonly used distributions (e.g. the Normal distribution), the elasticity is non-monotonic. However, for q such that at both c_1^* and c_0^* the elasticity is either decreasing or increasing, the same comparative static results as above apply. In particular, if the elasticity is concave in q (e.g. under the Normal distribution), prices are increasing in q for small buyers (i.e., those for which c_1^* and c_0^* lay on the region of increasing elasticity). In contrast, prices are decreasing in q for large buyers (i.e., those for which c_1^* and c_0^* lay on the region of decreasing elasticity). Hence, the medium-sized consumers are those who pay the highest prices: they are not small enough to benefit from signaling a low search cost when they participate in search, and they are not large enough to benefit from signaling high gains from search.

Figure 1 provides graphical examples of expected prices conditional on search under alternative distributional assumptions. One can see that prices follow a non-monotonic pattern in the case of the Normal distribution, being highest for medium-sized buyers. Prices are always decreasing in

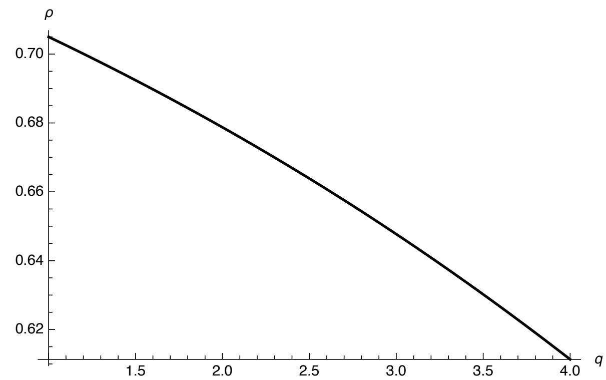
[33] There are several log-concave distribution functions for which $\varepsilon(c)$ is decreasing, e.g. the Exponential function $G(c) = \left(1 - e^{-c}\right)$ or the Pareto distribution $G(c) = 1 - \frac{1}{(c+1)^{\alpha}}$. The family of functions $G(c) = c^{\varepsilon}$ with $\underline{c} = 0$ have constant elasticity.

[34] This discussion is reminiscent of the literature in Public Finance that deals with the elasticity of earnings with respect to the tax rate over the distribution of income. See Saez (2001).

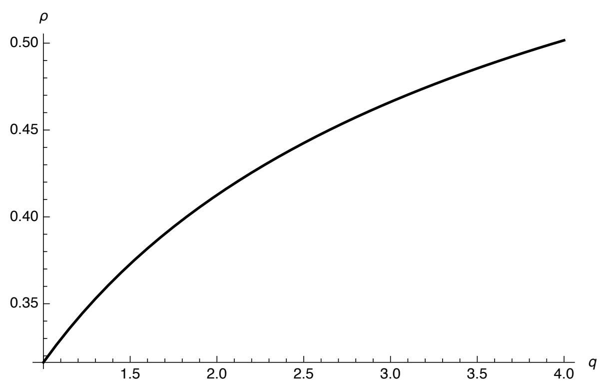
Figure 1: Expected prices (conditional on search) as a function of size q



(a) Non-monotonic expected prices (conditional on search) for Normal distribution, $\{\mu = 0, \sigma = 1\}$



(b) Decreasing expected prices (conditional on search) for Exponential distribution, $\{\beta = 1\}$



(c) Increasing expected prices (conditional on search) for Uniform distribution, $\{\underline{c} = -1, \overline{c} = 4\}$

(d) Flat expected prices (conditional on search) for Uniform distribution, $\{\underline{c} = 0, \overline{c} = 1\}$

Notes: Normal distribution as $N(\mu, \sigma)$. Exponential distribution parameterized as $CDF(c, \beta) = 1 - e^{\frac{c}{\beta}}$. Uniform distribution in range $U \sim [\underline{c}, \overline{c}]$.

size under the exponential distribution, due to the elasticity being everywhere decreasing. Finally, for the case of the uniform distribution, prices are increasing in size in the presence of shoppers, but they are constant otherwise. Indeed, when search costs are uniformly distributed between -1 and 4, i.e., with a 20% mass of shoppers, larger consumers face higher expected prices. Instead, when search costs are uniformly distributed between 0 and 4 all consumers pay the same price.

We can use the model to shed light on a related question: the likelihood with which large and small buyers engage in search, $G(c_0^*)$. Taking the derivative with respect to q,

$$\frac{\partial G(c_0^*)}{\partial q} = g(c_0^*)\left(\frac{c_0^*}{q} + \frac{\partial c_0^*}{\partial \rho^*}\frac{\partial \rho^*}{\partial q}\right). \tag{2}$$

The likelihood of search is increasing in c_0^*, which in turn is increasing in q and decreasing in ρ^*. Thus, for given ρ, larger buyers participate in search more often relative to the smaller ones. However, c_0^* also depends on q through ρ^*. If ρ^* is non-increasing in q (which according to Proposition 4 occurs if $\varepsilon(c_1^*) \geq \varepsilon(c_0^*)$), then equation (2) is unambiguously positive: larger buyers engage in search more often both because they gain more from search, but also because sellers compete more to serve them. In turn, this implies that larger buyers are unambiguously better-off, relative to the smaller ones.

Matters are not as clear when $\varepsilon(c_1^*) < \varepsilon(c_0^*)$. If ρ^* is increasing in q, the sign of equation (2) is *a priori* ambiguous (the first term in parenthesis is positive while the second one is negative). However, a simple logic shows that equation (2) cannot be negative: a necessary condition for smaller buyers to obtain lower prices conditional on search is that their participation decisions signal sufficiently low search costs; for this to be the case, small buyers must engage in search less often. Hence, $\partial\rho^*/\partial q$ positive (i.e., conditional on search, small buyers obtain lower prices) would contradict $\partial G(c_0^*)/\partial q$ negative (i.e., small buyers engage in search more often), and *viceversa*. It follows that $\partial G(c_0^*)/\partial q$ must be positive, regardless of the shape of G.

Therefore, if ρ^* is increasing in q, the relationship between the buyer's size and her (ex-ante) utility remains ambiguous: even when smaller buyers obtain lower expected prices once they search, they need not be better off as compared to the large buyers as they engage in search less often.

4 The effects of reducing search costs

In this section we want to understand how a reduction in search costs affects the various consumers. In particular, (i) how does a reduction in search costs affect prices? and (ii) how does it affect the relative prices of the large buyers relative to those of the small ones?

For this purpose, we perform comparative statics of prices with respect to search costs. We parametrize the search cost distribution as $G(c;\lambda)$, where a higher λ is associated with a smaller search cost.

We first focus on the comparative statics of expected prices conditional on search. Changing the search cost distribution through changes in λ creates two countervailing effects. First, conditional on search, an increase in λ reduces all buyers' (ex-ante) expected search costs, thus making it more likely that they solicit a second quote. However, since this also makes the first quote less costly, participation decisions are less informative of the buyer's realized search cost. The first effect pushes prices down, while the second effects pushes prices up. Which of the two effects dominates depends on whether the reduction in search costs affects more the lower deciles of the search cost distribution (which affect the probability of a second quote) or the upper ones (which affect participation decisions). For instance, for $\lambda < \lambda'$, suppose that $G(c_1;\lambda) < G(c_1;\lambda')$ but $G(c_0;\lambda) \geq G(c_0;\lambda')$. Then, from equation (1) the equilibrium ρ^* goes down and so prices are reduced. The contrary is true if $G(c_1;\lambda) \geq G(c_1;\lambda')$ but $G(c_0;\lambda) < G(c_0;\lambda')$. Thus, whether a reduction in search costs triggers a reduction or an increase in the expected price conditional on search crucially depends on how λ affects the search cost distribution.

To obtain formal predictions, we assume that λ shifts the distribution function upwards in a First Order Stochastic Dominance sense, i.e. $\partial G(c; \lambda) / \partial \lambda > 0$ for all λ.[35] We consider two cases, with λ entering the search cost distribution either (i) additively, $G(c+\lambda)$, or (ii) multiplicatively, $G(\lambda c)$.

Proposition 5 *Assume $G(c+\lambda)$, with $\lambda \geq 0$. A reduction in search costs (i.e., an increase in λ) leads to lower prices if and only G is log-concave (log-convex).*

When λ enters additively, an increase in λ shifts in the search density in a parallel fashion.[36] If the distribution of search costs is log-concave (log-convex), the impact on the lower deciles of the distribution is stronger (weaker) than on the higher deciles. Hence, prices conditional on search go down (up) for all buyers, regardless of their size.

Since there is a large family of distribution functions that are log-concave,[37] we focus our discussion on this case. As lower search costs reduce expected prices conditional on search, the threshold c_0^* goes up. This, together with the shift in the search density, increases the probability of engaging in search $G(c_0^* + \lambda)$. Overall, both the price and the participation effects lead to lower (ex-ante) expected prices. However, the extent of the price reduction need not be uniform across all buyers, an issue that depends on the shape of G. For instance, if search costs are normally distributed, a reduction in search costs implies weak price reductions on those buyers whose search cost thresholds lay on the tails of the distribution (i.e., either the very large or the very small consumers), while it leads to stronger price reductions for the medium-sized consumers.

Proposition 6 *Assume $G(\lambda c)$, with $\lambda \geq 1$. A reduction in search costs (i.e., an increase in λ) leads to lower prices only for those buyers for whom $\varepsilon(\lambda c_1^*) \geq \varepsilon(\lambda c_0^*)$.*

In contrast, when λ enters multiplicatively, whether a reduction in search costs leads to lower or higher prices depends on the shape of the search cost distribution. In particular, since the effect of reducing search costs is analogous to that of increasing size (note that q also enters multiplicatively in the expressions for the critical search thresholds), the same logic as in the previous section applies. Namely, a reduction in search costs reduces prices (conditional on search) if and only if the search cost distribution is more elastic at c_1^* than at c_0^*. Accordingly, conditional on search, a proportional reduction in search costs may lead to price reductions for all consumers (e.g., if costs are exponentially distributed), it might lead to price increases for all consumers (e.g., in the presence of shoppers, if costs are uniformly distributed), or it might have no effect on prices at all (e.g., in the absence of shoppers, if search costs are uniformly distributed). Interestingly, a proportional reduction in search costs may have a non-monotonic effect on prices (conditional on search).

[35] Alternatively, one could parametrize it as a function of β where $\lambda = 1/\beta$. In this case, $\partial G(c; \beta) / \partial \beta > 0$ so that an increase in β would be equivalent to an increase in search costs. For instance, β could be interpreted as the mean of the search cost distribution.

[36] For instance, if search costs are normally distributed, search cost distributions with higher λ have lower means.

[37] As it is well known, the family of log-concave distribution functions is very large, including the normal, exponential, uniform, logistic, log-normal and Pareto distributions, among others.

For instance, if costs are normally distributed, a reduction in search costs may lead to price increases for the small consumers (whose critical thresholds lay on the region with increasing elasticities), but to price reductions for the large ones. However, this does not mean that the small buyers are worse off, as the reduction in search costs also implies that they will engage in search more often. As for large buyers, a reduction in search costs makes them unambiguously better off, as they engage in search more often and, once they search, they obtain lower expected prices.

In order to illustrate the above theoretical findings, Figure 2 plots expected prices conditional on search for alternative search cost distributions, under both additive and multiplicative shifts. One can see that additive shifts that reduce search costs always imply lower expected prices (conditional on search) for all consumers, irrespective of their size. In the Figure, the solid line representing the baseline equilibrium is always above the short dashed line representing equilibrium prices following the parallel reduction in search costs. Also note that an additive shift can affect the properties of the distribution. For example, in the case of the exponential distribution, such a shift creates a mass of shoppers that implies that the elasticity is no longer monotonically decreasing. In turn, after the cost shock, prices are no longer monotonically decreasing with size.

In contrast, a multiplicative transformation of the search cost distribution that reduces search costs does not necessarily reduce expected prices for all of buyers. For example, when search costs are uniformly distributed in the range $[-1, 4]$ (a case with monotonically increasing elasticities), prices go up for all buyers following the reduction in search costs. When search costs are normally distributed, expected prices increase for smaller buyers (as their search thresholds fall in the region with increasing elasticities) but decrease for larger buyers (for whom the relevant elasticities are decreasing).

5 Uniform prices *versus* price discrimination

In this section we characterize equilibrium prices when sellers have to quote the same per-unit prices to all the buyers, irrespective of their size. The function $H(q)$ is introduced to denote the distribution of buyers in the market.

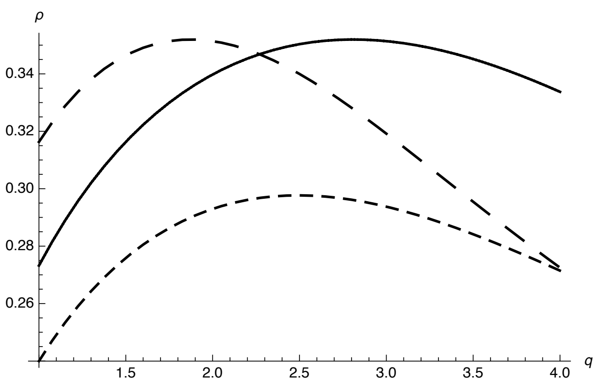
When price discrimination is not possible, firms choose their price quotes according to a mixed strategy no longer conditioned on the buyer's size. Let $\rho(q)$ denote the probability with which a buyer of size q searches once only. A seller's ex-ante expected profits from pricing at b are given by,

$$\pi(b) = b \int \left[\frac{\rho(q)}{2} + (1 - \rho(q))(1 - F(b)) \right] G(c_0(q))\, q dH(q).$$

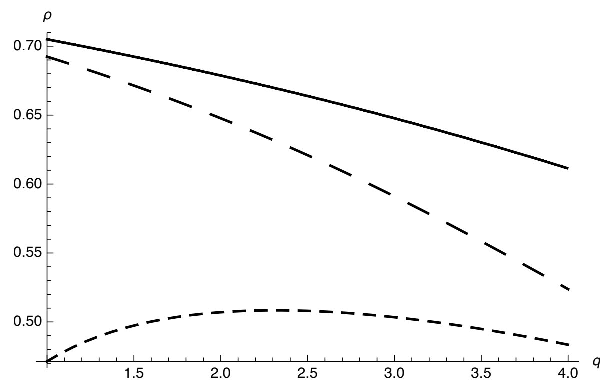
Profits are now weighted by the distribution of buyers' types in the population, $H(q)$, as well as by the probability that they engage in search, i.e., the endogenous object $G(c_0(q))$.

As shown in the following Proposition, there exists a symmetric equilibrium quote distribution which is analogous to the one in Proposition 1. The sole difference is that ρ is now replaced by $\tilde{\rho}$, which reflects the weighted average of the different values of $\rho(q)$ across the population of buyers

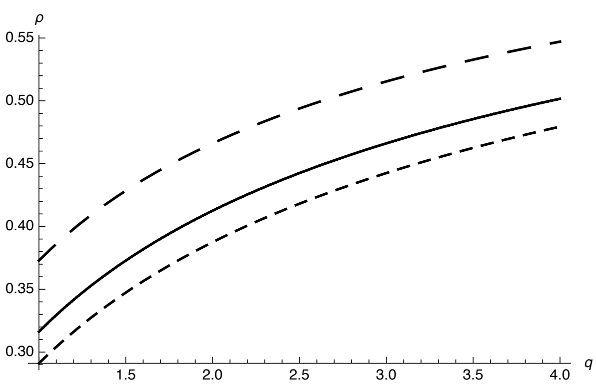
Figure 2: Expected prices with shifts in the search distribution



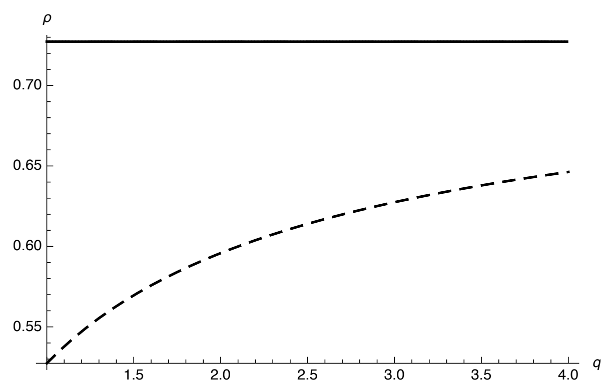
(a) Normal distribution, $\{v = 1, \mu = 0, \sigma = 1\}$



(b) Exponential distribution, $\{v = 1, \beta = 1\}$



(c) Uniform distribution, $\{v = 1, \underline{c} = -1, \overline{c} = 4\}$



(d) Uniform distribution, $\{v = 1, \underline{c} = 0, \overline{c} = 1\}$

Notes: The solid line represents baseline expected prices (conditional on search) as a function of size q. The large dashed line shifts the distribution multiplicatively, $G(\lambda c)$, with $\lambda = 1.5$. The small dashed line shifts the distribution additively, $G(c + \lambda)$, with $\lambda = 0.2$. One can see that an additive shift in the distribution always reduces prices (solid line vs small dashed line). A multiplicative shift has ambiguous effects to prices.

who search. In other words, firms price *as if* they were facing the "average" buyer.

Proposition 7 *With uniform prices, there is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by*

$$F(b) = 1 - \frac{1}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\frac{1-b}{b},$$

with compact support $b \in \left[\frac{\tilde{\rho}}{2-\tilde{\rho}}, 1\right]$, *where*

$$\tilde{\rho} = \frac{\int \rho(q)\, G(c_0(q))\, q dH(q)}{\int G(c_0(q))\, q dH(q)} \in (0,1). \tag{3}$$

Even if all sellers quote the same prices to all buyers, the buyer-specific probability of searching once, $\rho(q)$, need not be the same across all sizes. In turn, this might introduce differences in the prices effectively paid by each buyer.[38]

For a given pricing behavior of the sellers, buyers' search decisions do not depend on whether other consumers face similar or dissimilar prices. Thus, search behavior is not directly affected by whether price discrimination is allowed or not. The effect is only indirect, to the extent that sellers' pricing behavior changes when price discrimination is no longer allowed. Accordingly, Proposition 2 remains unchanged. The sole difference is that the critical thresholds for the search strategy are now a function of the average $\tilde{\rho}$ rather than being buyer-specific. To make this explicit, we now denote the search thresholds as $\tilde{c}_1$ and $\tilde{c}_0$.

The equilibrium values of $\rho(q)$, one for each buyer type, are now found as the solution to the following system of equations, one for each q:

$$\rho^*(q) = 1 - \frac{G(\tilde{c}_1^*)}{G(\tilde{c}_0^*)} \in (0,1). \tag{4}$$

Each of these conditions is analogous to (1), with two main differences. First, there are now as many conditions as buyer sizes. And second, the right hand side of the equation now depends on the average $\tilde{\rho}$ rather than on $\rho(q)$. This implies that the search intensities that we found in the equilibrium with price discrimination cannot be an equilibrium in the game with uniform prices, unless the search intensities of all buyers are identical, i.e., unless $\rho(q) = \tilde{\rho}$ for all q. From the analysis in the previous section we know this is the case for distribution functions $G(c)$ with constant elasticity, e.g. the uniform distribution with $\underline{c} = 0$.

This has an important implication: when search costs are uniformly distributed with $\underline{c} = 0$, prices and search intensity are the same with and without price discrimination. For all other distributions, there does not exist equilibria such that all buyers use the same search intensity.

[38] Interestingly, note that the distribution of sizes (which is exogenous) affects $\tilde{\rho}$, and hence the prices charged to all buyers in the no-discrimination case. In this sense, changes in the distribution of buyers' size (e.g. following a merger between buyers) affect the degree of competition among sellers, thus affecting all buyers in the industry. The effect might be positive or negative depending on whether the average $\tilde{\rho}$ decreases or increases after the merger, an issue which will depend on the relative sizes of the merging parties.

Therefore, equilibrium prices change when discrimination is no longer allowed. Analogously to Proposition 3, there exists a SPNE in the game with uniform prices.

Proposition 8 *With uniform prices, there exists a symmetric SPNE in which sellers price as stated in Proposition 7 and buyers search as stated in Proposition 2. Conditional on participating in search, the probability that the buyer asks for one quote is given by the solution to equation (4).*

Under uniform prices, the comparative statics of search intensity and expected prices with respect to buyers' size are analogous to those in Proposition 4.

Proposition 9 *In equilibrium, the expected price conditional on search, $\rho^*(q)$, is (i) decreasing in q if $\varepsilon(\tilde{c}_1^*) > \varepsilon(\tilde{c}_0^*)$; (ii) increasing in q if $\varepsilon(\tilde{c}_1^*) < \varepsilon(\tilde{c}_0^*)$; and (iii) constant in q if $\varepsilon(\tilde{c}_1^*) = \varepsilon(\tilde{c}_0^*)$.*

Proposition 9 highlights that, even though all buyers face the same quote distribution, they search differently depending on their size. As a result, those buyers who solicit relatively more quotes pay lower prices. However, these are not necessarily the larger buyers. Just as before, the comparison of the elasticities at the search thresholds, $\tilde{c}_1^*$ and $\tilde{c}_0^*$, determines whether larger or smaller buyers solicit more quotes, thus giving rise to expected prices (conditional on search) that are either decreasing or increasing in consumers' size.

We are now ready to understand the effects of allowing for price discrimination by comparing the results obtained in this section to those in section 2.3. For the sake of exposition, we add the subscripts d and u to distinguish the cases with discriminatory or uniform prices, respectively. We first focus on comparing the search intensity of a buyer with size q.

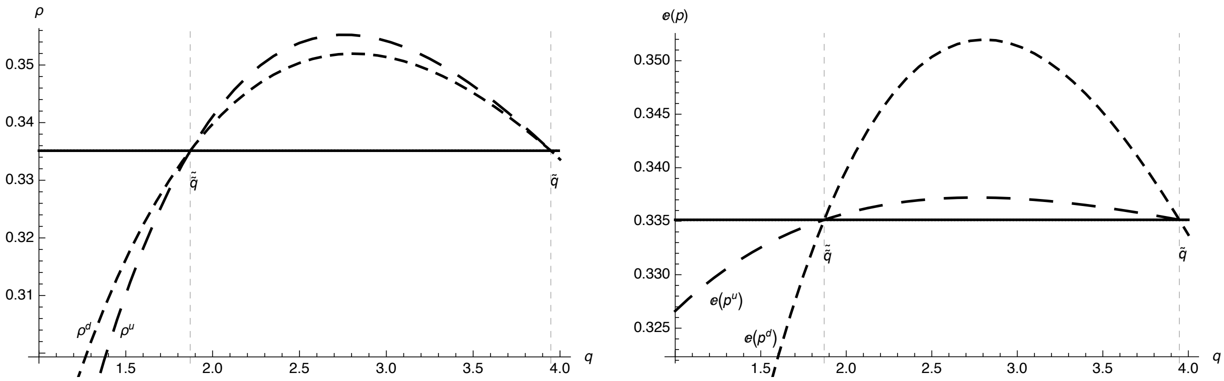
Proposition 10 *(i) If G has a decreasing elasticity, there exists $\widetilde{q}$ such that $\rho^u(q) \geq \rho^d(q) \geq \widetilde{\rho}^u$ for $q \leq \widetilde{q}$ (small buyers) and $\rho^u(q) < \rho^d(q) < \widetilde{\rho}^u$ for $q > \widetilde{q}$ (large buyers); and (ii) if G has an increasing elasticity, there exists $\widetilde{\widetilde{q}}$ such that the reverse ordering holds.*

Consider the case of decreasing elasticities (part (i) of the statement) and let us refer to a small (large) buyer as one with a size below (above) $\widetilde{q}$.[39] With uniform prices, small consumers benefit from being pooled with larger buyers. Hence, they can obtain lower prices than under discriminatory prices with no need to search as much (i.e., for small consumers, the probability of searching once is higher under uniform prices, $\rho^u(q) \geq \rho^d(q)$). On the contrary, larger buyers do not obtain a price advantage for being large and thus need to search more relative to the case with price discrimination (i.e., for large buyers, $\rho^u(q) < \rho^d(q)$). The opposite holds true when the distribution function has increasing elasticities as in this case the large buyers gain from being pooled with the small ones.

As stated in the following Proposition, this logic translates into the ranking of expected prices conditional on search across the two pricing policies. In particular, since the price effect is first

[39]From the previous section, recall that the exponential and the Pareto distributions, among others, depict monotonically decreasing elasticities. The same applies to the Normal distribution for large qs.

Figure 3: Comparison between uniform and discriminatory pricing



(a) Probability of soliciting one quote conditional on search, Normal distribution $\{\mu = 0, \sigma = 1\}$

(b) Equilibrium expected price conditional on search, Normal distribution $\{\mu = 0, \sigma = 1\}$

Notes: Comparison of search intensities and equilibrium prices between uniform and discriminatory prices under the Normal distribution. The solid line represents average equilibrium search behavior ($\widetilde{\rho}$) and average expected prices under uniform prices, $\widetilde{\rho}^u$. The long dash line represents these same objects but conditional on q, i.e., $\rho(q)$ and $E[p^u(q)]$. The short dash line represents equilibrium outcomes under price discrimination, i.e., $\rho^d(q)$ and $E[p^d(q)]$. Example computed with four types with equal weight and $q \in \{1, 2, 3, 4\}$.

order relative to the search effect, the prices paid by the small buyers are reduced, while those of the large buyers are increased, when prices have to be uniform and the search cost distribution function depicts decreasing elasticities. In turn, the lower prices faced by the small buyers induce them to engage in search more often, which further strengthens the price reduction. The reverse ranking holds under increasing elasticities.

Proposition 11 *(i) Suppose that G has a decreasing elasticity, then with uniform prices, the expected price paid by the small (large) buyers conditional on search are lower (greater) than under price discrimination; (ii) the reverse ordering holds if G has an increasing elasticity.*

Figure 3 illustrates the results from Propositions 10 and 11 when search costs are normally distributed: the left panel represents search intensities and the right panel represents expected prices (conditional on search). Let us focus on the range with increasing elasticity (from low qs up to $\widetilde{\widetilde{q}} \approx 2$). The probability that a buyer with $q < \widetilde{\widetilde{q}}$ asks for one quote only is lower under uniform prices (long dash) than under price discrimination (short dash). However, such buyer ends up paying more than under price discrimination due to the cross-subsidization to other buyer types. The mirror image is obtained for the range with a decreasing elasticity (above $\widetilde{q} \approx 3.8$).

One can also see that the price dispersion across buyers' types is naturally smaller under uniform prices, due to the pooling of the price distribution. The attenuation in dispersion is particularly marked for those consumers on the tails of the size distribution. Under the Normal distribution, such differences get wider as one moves away from the consumer that is equally well off with uniform or with discriminatory pricing.

6 Conclusions

In this paper we have built a model of search with third-degree price discrimination that sheds light on the determinants of price heterogeneity across heterogeneous buyers, an issue that has become increasingly relevant for competition policy.

In our model, size differences introduce heterogeneity in buyers' willingness to search, and this opens up the scope for price discrimination. In particular, sellers charge lower prices to those buyers with a higher willingness to search. The key to assessing buyer power is thus to understand whether large buyers have higher willingness to search (conditional on search) as compared to the small ones. In this paper we have shown that this issue ultimately depends on the shape of the search cost distribution function.

In particular, we have identified a simple condition to predict the relationship between prices and buyers' size: if the elasticity of the search cost distribution is decreasing (increasing) over the relevant range, then larger buyers pay less (more) then the smaller ones. If search costs are normally distributed, the elasticities depict an-inverted U-shaped. Hence, prices conditional on search are also non-monotonic in size, with medium-size consumers paying more than either the small or the large ones. Intuitively, the medium-sized consumers do not benefit as much as the small ones from signaling low search costs when they engage in search, nor do they benefit as much as the large ones from signaling higher gains from search. The effects of size on expected prices also determine the likelihood of search in the first place, and thus the distribution of the size of buyers who search.

A reduction in search costs does not affect all buyers uniformly. In particular, if cost shifts are additive (e.g. if, in the case of the Normal distribution, additive cost shocks reduce the mean but leave the rest of the distribution unchanged), prices go down for all buyers (under log-concave search cost distributions), but the effect is more pronounced for the large ones. However, if cost shifts are multiplicative, a reduction in search costs has the same impact as an increase in the buyer's size, thus leading to the same results as discussed above. In particular, if search costs are normally distributed, then small consumers (conditional on search), face higher prices when a multiplicative shift drives search costs down. However, this does not mean that they are necessarily worse off, as the reduction in search costs also implies that they will engage in search more often.

Our analysis provides predictions regarding the effects of price discrimination on the various consumers as well as on overall welfare. Since these predictions rely on the elasticities of the search cost distribution, which need not coincide across all markets, it is simply not possible to provide a single answer to the general question of whether a ban on price discrimination would affect prices. Inexorably, the answer has to rely on industry specific studies that shed light on consumers' search behavior as well as on the distribution of the various consumers' types in the market. This paper has simply demonstrated that a ban on price discrimination aimed at protecting the small consumers need not always achieve the intended goal.

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A Appendix: Proofs

Proof of Lemma 1. Suppose $\rho = 1$. Then, the seller knows that it is a monopolist and hence charges the reservation price. However, it would leave no surplus for the buyer, so her participation constraint would not be satisfied. Hence, $\rho = 0$ cannot be part of an equilibrium in which the buyer has decided to participate. Suppose $\rho = 0$. Then, there is Bertrand competition with both sellers quoting prices equal to marginal cost. However, knowing that all sellers choose the same quote, the buyer would have incentives to deviate and search less in order to save on search costs. Hence, $\rho = 0$ cannot be part of an equilibrium. As a consequence, in equilibrium, we must have $\rho \in (0, 1)$.

■

Proof of Proposition 1. Let sellers choose the distribution of quotes $F(b)$ over the interval $\left[\underline{p}, \overline{b}\right]$. Standard arguments imply that this distribution must be atomless. In particular, if sellers put mass on a given quote, there would be a positive probability of a tie. If such a quote is above marginal costs, each seller would be better off by slightly reducing its quote below that level: this would have a minor effect on its profits if the buyer has asked for one quote only, but would guarantee that the seller makes the sale whenever the buyer has asked for two quotes. Putting mass at marginal cost cannot be part of an equilibrium either, given that each seller would be able to make the sale at a higher price whenever the buyer has asked for one quote only.

Seller i's profits from quoting a price b when rivals are choosing quotes according to $F(b)$ are given by:[40]

$$\pi(b) = bq\left(\frac{\rho}{2} + (1-\rho)(1-F(b))\right).$$

[40] Results would be the same if we instead computed the firm's expected profits using probabilities conditional on having received a quote request. Let $x = \frac{\rho/2}{1-\rho/2}$ be the conditional probability of receiving a quote request from a buyer who has only asked for one quote only. Expected profits would thus be written as $\pi(b) = bq(x + (1-x)(1-F(b)))$. The equilibrium price distribution would be $F(b) = 1 - \frac{x}{1-x}\frac{1-b}{b}$, with b in the compact support $p \in [x, 1]$. While both formulations are equivalent, we believe that using the unconditional probability ρ allows for a more intuitive interpretation of the results.

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi\left(b;q\right)=\overline{\pi}\text{ for all }b\in\left[\underline{p},\overline{b}\right].$$

Furthermore, as $\pi\left(\overline{b}\right)=\overline{b}q\frac{\rho}{2}$ is increasing in $\overline{b}$, it follows that $\overline{b}=1$. Hence, $\overline{\pi}=q\frac{\rho}{2}$. From $\pi\left(\underline{p}\right)=\underline{p}q\left(\frac{\rho}{2}+(1-\rho)\right)=\overline{\pi}=q\frac{\rho}{2}$ it also follows that $\underline{p}=\frac{\rho}{2-\rho}$. Accordingly, the support of the equilibrium mixed strategy is $b\in\left[\frac{\rho}{2-\rho},1\right]$.

To obtain the equilibrium quote distribution, from $\pi\left(b\right)=q\frac{\rho}{2}$, it follows that

$$F\left(b\right)=1-\frac{1}{2}\frac{\rho}{1-\rho}\frac{1-b}{b},$$

with density

$$f\left(b\right)=\frac{1}{2}\frac{\rho}{1-\rho}\frac{1}{b^2}.$$

For $\rho\to1$, the equilibrium collapses to the reservation price, whereas for $\rho\to0$, the equilibrium collapses to marginal costs. Sellers' expected profits when quoting a price to a buyer of size q are thus $q\rho$. ■

Proof of Lemma 2 . We first derive the distribution of the price paid by the buyer (conditional on search), which we denote as F_p, using the quote distribution characterized above. When the buyer has searched n times, the distribution is $(1-(1-F(b))^n)$. Hence, given that buyers only get one quote with probability ρ, one finds the distribution the expected price as,

$$\begin{aligned}F_p\left(b\right) &= \rho F(b)+(1-\rho)\left(1-(1-F(b))^2\right)\\ &= \frac{\left(\rho-b\left(2-\rho\right)\right)\left(\rho-b\left(2+\rho\right)\right)}{\left(2b\right)^2\left(1-\rho\right)},\end{aligned}$$

with density

$$f_p\left(b\right)=\frac{\rho^2}{2b^3\left(1-\rho\right)}.$$

Note that the price distribution $F_p\left(b\right)$ is decreasing in ρ,

$$\frac{\partial F_p\left(b\right)}{\partial\rho}=-\frac{\rho\left(2-\rho\right)\left(1-b\right)\left(1+b\right)}{4b^2\left(1-\rho\right)^2}<0.$$

With this expression we can compute the expected price paid by the buyer (conditional on search) as

$$\int_{\underline{b}}^{\overline{b}}bdF_p\left(b\right)=\rho.$$

■

Proofs of Proposition 2. A buyer who searches once or twice derives utility

$$\begin{aligned} u_1 &= \left(1-\int_{\underline{b}}^{\overline{b}} bdF(b)\right)q-c \\ u_2 &= \left(1-\int_{\underline{b}}^{\overline{b}} 2b(1-F(b))\,dF(b)\right)q-2c, \end{aligned}$$

with $F(b)$ and $[\underline{p},\overline{b}]$ as characterized in Proposition 1. The functions $u_1(c)$ and $u_2(c)$ are linear in c, with constant slope -1 and -2, respectively, and $u_2(0)>u_1(0)>0$. It follows that there exist $0<c_1<c_0<q$ such that (i) $0>u_1(c)>u_2(c)$ for $c>c_0$; (ii) $u_1(c)\geq\max\{u_2(c),0\}$ for $c\in(c_1,c_0]$ and (iii) $u_2(c)\geq u_1(c)>0$ for $c\leq c_1$. More specifically, c_0 and c_1 are implicitly defined by $u_1(c_0)=0$ and $u_1(c_1)=u_2(c_1)$. We next derive explicit formula, using the equilibrium quote distribution characterized in Proposition 1.

Conditional on observing one quote, the expected utility is

$$\begin{aligned} u_1 &= \left(1-\int_{\underline{b}}^{\overline{b}} bdF(b)\right)q-c \\ &= q\left(1-\frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right)-c. \end{aligned}$$

Hence, to be indifferent between searching or not:

$$c_0=q\left(1-\frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right).$$

The threshold c_0 is decreasing in ρ, with $\lim_{\rho\to0}c_0=q$ and $\lim_{\rho\to1}c_0=0$. For future reference,

$$\lim_{\rho\to1}\frac{\partial c_0}{\partial\rho}=-q.$$

Conditional on observing two quotes, the expected price is the minimum of the two, so the utility is

$$\begin{aligned} u_2 &= \left(1-\int_{\underline{b}}^{\overline{b}} 2b(1-F(b))\,dF(b)\right)q-2c \\ &= q\left(1+\frac{\rho^2}{2(1-\rho)^2}\left(2-\frac{2}{\rho}+\ln\frac{2-\rho}{\rho}\right)\right)-2c. \end{aligned}$$

Equating the two utilities and solving for c:

$$c_1=q\frac{\rho}{1-\rho}\left(\frac{1}{2(1-\rho)}\ln\left(\frac{2-\rho}{\rho}\right)-1\right)$$

The threshold c_1 is concave in ρ,[41] with $\lim_{\rho\to 0} c_1 = \lim_{\rho\to 1} c_1 = 0$. For future reference,

$$\lim_{\rho\to 1}\frac{\partial c_1}{\partial \rho} = -\frac{q}{3}.$$

Taking the difference between the two thresholds:

$$c_0 - c_1 = q\frac{2-2\rho-\rho\left(2-\rho\right)\ln\left(\frac{2-\rho}{\rho}\right)}{2\left(1-\rho\right)^2}.$$

The difference is decreasing in ρ, with $\lim_{\rho\to 0}\left(c_0-c_1\right) = q$ and $\lim_{\rho\to 1}\left(c_0-c_1\right) = 0$. For future reference,

$$\frac{\partial c_0}{\partial \rho} - \frac{\partial c_1}{\partial \rho} = q\frac{2-2\rho-\ln\left(\frac{2-\rho}{\rho}\right)}{\left(1-\rho\right)^3} < 0. \tag{5}$$

■

Proof of Proposition 3. We need to show that there exists a solution to (1) in $(0,1)$. To show that condition (1) has a solution, note that $\lim_{\rho\to 0} c_1 = 0$ and $\lim_{\rho\to 0} c_0 = q$. Hence, when $\underline{c} < 0$

$$\lim_{\rho\to 0}\left(1-\frac{G\left(c_1\right)}{G\left(c_0\right)}\right) = 1-\frac{G\left(0\right)}{G\left(q\right)} \in \left(0,1\right),$$

whereas when $\underline{c} = 0$,

$$\lim_{\rho\to 0}\left(1-\frac{G\left(c_1\right)}{G\left(c_0\right)}\right) = 1.$$

Furthermore, $\lim_{\rho\to 1} c_1 = \lim_{\rho\to 1} c_0 = 0$, implying that $G\left(c_1\right) = G\left(c_0\right)$. Hence, when $\underline{c} < 0$

$$\lim_{\rho\to 1}\left(1-\frac{G\left(c_1\right)}{G\left(c_0\right)}\right) = 0.$$

When $\underline{c} = 0$, both $G\left(c_1\right)$ and $G\left(c_0\right)$ take the value 0 for $\rho \to 1$, so $\lim_{\rho\to 1}\frac{G(c_1)}{G(c_0)}$ is undefined. Applying l'Hôpital,

$$\begin{aligned}\lim_{\rho\to 1}\left(1-\frac{G\left(c_1\right)}{G\left(c_0\right)}\right) &= 1-\lim_{\rho\to 1}\frac{g\left(c_1\right)\frac{\partial c_1}{\partial \rho}}{g\left(c_0\right)\frac{\partial c_0}{\partial \rho}} \\ &= 1-\frac{g\left(0\right)}{g\left(0\right)}\frac{q/6}{q/2} \\ &= \frac{2}{3}.\end{aligned}$$

Under regularity conditions on G that ensure continuity, note that: (i) for $\underline{c} = 0$, the function takes the value 1 for $\rho \to 0$ and a lower value for $\rho \to 1$, and (ii) for $\underline{c} < 0$, the function takes a strictly positive value for $\rho \to 0$ and zero for $\rho \to 1$. Hence, in both cases, the function must cross the 45

[41] The maximum is achieved at $\rho = 0.365$.

degree line at some $\rho^* \in (0,1)$. Hence, there exists an interior solution to (1).

[Uniqueness] A sufficient condition for equilibrium uniqueness is that G is log-convex as in this case the right hand side of (1) is everywhere decreasing in ρ. Hence, it crosses the 45-degree line only once. In detail, the general derivative for the function is

$$\frac{\partial\left(1-\frac{G(c_1)}{G(c_0)}\right)}{\partial \rho}=\frac{g\left(c_0\right) G\left(c_1\right) \frac{\partial c_0}{\partial \rho}-g\left(c_1\right) G\left(c_0\right) \frac{\partial c_1}{\partial \rho}}{\left(G\left(c_0\right)\right)^2}.$$

Since the denominator is always positive, we focus on the numerator. It will be negative as long as,

$$\frac{g\left(c_0\right)}{G\left(c_0\right)} \frac{\partial c_0}{\partial \rho}<\frac{g\left(c_1\right)}{G\left(c_1\right)} \frac{\partial c_1}{\partial \rho}. \tag{6}$$

Using our previous results in the proof of Proposition 5,

$$\frac{\partial c_0}{\partial \rho}<0 \text{ and } \frac{\partial c_0}{\partial \rho}<\frac{\partial c_1}{\partial \rho}.$$

Hence, condition (6) is satisfied if G is log-convex, as this implies

$$\frac{g\left(c_0\right)}{G\left(c_0\right)}>\frac{g\left(c_1\right)}{G\left(c_1\right)}.$$

If G is log-concave, the right hand side of (1) is decreasing in ρ up to $\rho \leq 0.365$ as in this case $\partial c_1/\partial\rho > 0$. However, for $\rho > 0.365$ the right hand side of (1) can eventually become positively sloped so that we cannot resort to the same argument to show uniqueness.

A less stringent sufficient condition for uniqueness is that the slope of the right hand side of (1) is below 1. This condition becomes

$$\frac{G\left(c_0\right)}{G\left(c_1\right)}>\frac{g\left(c_0\right)}{G\left(c_0\right)} \frac{\partial c_0}{\partial \rho}-\frac{g\left(c_1\right)}{G\left(c_1\right)} \frac{\partial c_1}{\partial \rho}.$$

If condition (6) is satisfied, the RHS of the above equation is negative so that the condition is always satisfied. If the RHS is positive (meaning that the schedule $1-G\left(c_1\right)/G\left(c_0\right)$ is positively sloped), the condition essentially requires that G is not too concave. ■

Proof of Proposition 4. For ρ^* to be decreasing (increasing) in q, the RHS of the equilibrium condition (1) must be decreasing (increasing) in q. This is the case if and only if

$$\frac{\partial\left(1-\frac{G(c_1)}{G(c_0)}\right)}{\partial q}=-\frac{c_1 G\left(c_0\right) g\left(c_1\right)-c_0 G\left(c_1\right) g\left(c_0\right)}{q\left(G\left(c_0\right)\right)^2}<0.$$

where we have used the fact that both c_1 and c_0 are linear in q so that their derivatives are simply c_1/q and c_0/q respectively. Since the denominator is positive, we focus attention on the numerator, which can be re-written as

$$G\left(c_0\right)G\left(c_1\right)\left[c_0\frac{g(c_0)}{G(c_0)}-c_1\frac{g(c_1)}{G(c_1)}\right]<0,$$

or using the expression for $\varepsilon\left(c\right)\equiv c\frac{g(c)}{G(c)}$,

$$G\left(c_0\right)G\left(c_1\right)\left[\varepsilon\left(c_0\right)-\varepsilon\left(c_1\right)\right]<0.$$

It follows that a sufficient condition for ρ^* to be decreasing (increasing) in q is that the term in brackets, evaluated at ρ^*, is negative (positive). Hence, as q goes up, the schedule crosses the 45-degree line at a smaller (higher) value of ρ. If the elasticity $\varepsilon\left(c\right)$ is constant, changes in q do not move the schedule, and hence the equilibrium remains unchanged. Note that if G is log-convex, then $\varepsilon\left(c_0\right)>\varepsilon\left(c_1\right)$ so that ρ^* is increasing in q. ■

Proof of Proposition 5. Consider additive shocks, $G\left(c+\lambda\right)$. For expected prices (conditional on search) to be decreasing in λ, the solution to $\rho=1-\frac{G(c_1+\lambda)}{G(c_0+\lambda)}$ must be decreasing in λ,

$$\frac{\partial\left(1-\frac{G(c_1+\lambda)}{G(c_0+\lambda)}\right)}{\partial\lambda}=-\frac{G\left(c_0+\lambda\right)g\left(c_1+\lambda\right)-G\left(c_1+\lambda\right)g\left(c_0+\lambda\right)}{\left(G\left(c_0+\lambda\right)\right)^2}.$$

The denominator is positive for G log-concave and negative for G log-convex. ■

Proof of Proposition 6. Consider multiplicative shocks, $G\left(\lambda c\right)$. The same logic as in the proof of Proposition 4 applies given that λ enters multiplicatively, just as q enters multiplicatively in the expressions of the thresholds c_0 and c_1. In detail, for expected prices (conditional on search) to be decreasing in λ, the solution to $\rho=1-\frac{G(\lambda c_1)}{G(\lambda c_0)}$ must be decreasing in λ,

$$\frac{\partial\left(1-\frac{G(\lambda c_1)}{G(\lambda c_0)}\right)}{\partial\lambda}=-\frac{c_1G\left(\lambda c_0\right)g\left(\lambda c_1\right)-c_0G\left(\lambda c_1\right)g\left(\lambda c_0\right)}{\left(G\left(\lambda c_0\right)\right)^2}<0.$$

Since the denominator is positive, we focus on the numerator, which can be re-written as:

$$\frac{G\left(\lambda c_0\right)G\left(\lambda c_1\right)}{\lambda}\left[\lambda c_0\frac{g(\lambda c_0)}{G(\lambda c_0)}-\lambda c_1\frac{g(\lambda c_1)}{G(\lambda c_1)}\right]<0,$$

or using the expression for $\varepsilon\left(c\right)\equiv c\frac{g(c)}{G(c)}$,

$$\frac{G\left(\lambda c_0\right)G\left(\lambda c_1\right)}{\lambda}\left[\varepsilon\left(\lambda c_0\right)-\varepsilon\left(\lambda c_1\right)\right]<0.$$

Hence, focusing on the term in brackets, the expression $1-\frac{G(\lambda c_1)}{G(\lambda c_0)}$ is decreasing in λ iff $\varepsilon\left(\lambda c_0\right)<\varepsilon\left(\lambda c_1\right)$. Since $c_1<c_0$, a sufficient condition is that the elasticity is decreasing in c. For constant $\varepsilon\left(c\right)$, the equilibrium condition does not change with λ. ■

Proof of Proposition 7. Similar arguments are those in Proposition 1 allow us to conclude that there does not exist an equilibrium in pure strategies.

Let seller i's profits from quoting a price b when the rival is choosing quotes according to $F(b;q)$ be given by:

$$\pi(b) = b \int q \left[\rho(q)/2 + (1-\rho(q))(1-F(b))\right] G(c_0(q))\, q dH(q).$$

Let $x = \int q\rho(q) G(c_0(q))\, dH(q)$ and $y = \int qG(c_0(q))\, dH(q)$ with $x < y$. Profits can be re-written as

$$\pi(b) = b\left(x + (y-x)(1-F(b))\right).$$

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi(b) = \overline{\pi} \text{ for all } b \in \left[\underline{p}, \overline{b}\right].$$

Furthermore, as $\pi\left(\overline{b}\right) = \overline{b}y$ is increasing in $\overline{b}$, it follows that $\overline{b} = 1$. Hence, $\overline{\pi} = x$. From $\pi\left(\underline{p}\right) = \underline{p}y = \overline{\pi} = x$ it also follows that $\underline{p} = \frac{x}{y}$. To obtain the equilibrium quote distribution, from $\pi(b) = x$, it follows that

$$F(b) = 1 - \frac{1}{2}\frac{x}{y-x}\frac{1-b}{b}\cdot$$

Renormalize it by defining

$$\tilde{\rho} \equiv \frac{\int \rho(q)\, qG(c_0(q))\, dH(q)}{\int qG(c_0(q))\, dH(q)} < 1$$

(i.e., $\tilde{\rho}$ is the weighted average ρ) so that

$$F(b) = 1 - \frac{1}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\frac{1-b}{b}\cdot$$

■

Proof of Proposition 8. Using the expression for $\tilde{\rho}$ above, and plugging it in the equilibrium condition,

$$\tilde{\rho} = \frac{\int \left(1 - \frac{G(\tilde{c}_1(q))}{G(\tilde{c}_0(q))}\right) G(c_0(q))\, qdH(q)}{\int G(c_0(q))\, qdH(q)}\cdot$$

With some algebra,

$$\begin{aligned}
\tilde{\rho} &= \frac{\int \left(G(\tilde{c}_0(q)) - G(\tilde{c}_1(q))\right) qdH(q)}{\int G(c_0(q))\, qdH(q)} \\
&= 1 - \frac{\int G(\tilde{c}_1(q))}{\int G(\tilde{c}_0(q))} dH(q)
\end{aligned}$$

The equilibrium conditions on the buyers' and sellers' sides can be combined into a single equilibrium equation:

$$\tilde{\rho} = 1 - \frac{\int G(c_1(q,\tilde{\rho}))}{\int G(c_0(q,\tilde{\rho}))} dH(q).$$

Therefore, one can solve the above equation to find the equilibrium $\tilde{\rho}$ by substituting in the values of $\tilde{c}_0(q)$ and $\tilde{c}_1(q)$ as a function of $\tilde{\rho}$, which is analogous to the case with price discrimination. ■

Proof of Proposition 9. The proof follows similar steps as the proof of Proposition 4, and it is therefore omitted. ■

Proof of Proposition 10. First, we want to show that there exists at least one $\widetilde{q}$ such that at $\widetilde{q}$, $\rho^d(\widetilde{q}) = \widetilde{\rho}^u = \rho^u(\widetilde{q})$. These ρ values are defined in equation equation (6) for $\rho^d(q)$, equation (3) for $\widetilde{\rho}^u$, and equation (4) for $\rho^u(q)$. The schedules $\widetilde{\rho}^u$ and $\rho^u(q)$ have to cross since $\widetilde{\rho}^u$ is the average of $\rho^u(q)$. Let $\widetilde{q}$ be the value(s) at which they cross. In turn, the RHS of the expressions for $\rho^d(q)$ and $\rho^u(q)$ are the same, so they are equal if we evaluate them at the same (q, ρ). It follows that $\rho^d(\widetilde{q}) = \widetilde{\rho}^u = \rho^u(\widetilde{q})$.

Second, for the case of decreasing elasticities, we want to show that if $q < \widetilde{q}$ then $\rho^u(q) > \rho^d(q) > \widetilde{\rho}^u$; while if $q > \widetilde{q}$ then $\rho^u(q) < \rho^d(q) < \widetilde{\rho}^u$. The reverse ordering is true for the case of increasing elasticities.

From our previous Propositions, since $\rho^u(q)$ and $\rho^d(q)$ are decreasing in q, $q < \widetilde{q}$ implies $\rho^u(q) > \widetilde{\rho}^u$ and $\rho^d(q) > \widetilde{\rho}^u$. Furthermore, whenever $1 - \frac{G(c_1)}{G(c_0)}$ is decreasing in ρ (as guaranteed by condition (6); see the proof of Proposition 3), the latter implies

$$\rho^u(q) = 1 - \frac{G\left(c_1(q, \widetilde{\rho}^u)\right)}{G\left(c_0(q, \widetilde{\rho}^u)\right)} > 1 - \frac{G\left(c_1(q, \rho^d)\right)}{G\left(c_0(q, \rho^d)\right)} = \rho^d(q).$$

The proof for $q > \widetilde{q}$ as well as the proof for the case with increasing elasticities are analogous, and therefore omitted. ■

Proof of Proposition 11. We show the results for the case of decreasing elasticities, as those for increasing elasticities are analogous. We want to show that expected prices conditional on search are larger under discriminatory pricing than under uniform pricing for $q < \widetilde{q}$. This comes directly from the previous result. Prices at $\widetilde{q}$ are the same under uniform and discriminatory pricing by construction, as $\widetilde{\rho}^u$ equals $\rho^u(q)$ and $\rho^d(q)$ at that point. Expected prices as a function of q are decreasing for both cases, but steeper for the discriminatory case. Since c_0 is decreasing in the expected price conditional on search, the likelihood of search is higher with uniform prices for buyers with $q < \widetilde{q}$, and so their (ex-ante) expected price is reduced. ■