Product Choice and Price Discrimination in Markets with Search Frictions*

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Abstract

In a seminal paper, Champsaur and Rochet (1989) showed that competing firms choose non-overlapping qualities so as to soften price competition at the cost of giving up profitable opportunities to price discriminate. We show that an arbitrarily small amount of search frictions is enough to rule out such equilibrium. Instead, there exists an equilibrium with overlapping qualities and full price discrimination. This is in contrast to other sources of market power (e.g. horizontal product differentiation), which have to be sufficiently strong in order to give rise to overlapping qualities. Search frictions increase prices and reduce consumers surplus for given quality choices, but they can also lead to lower prices and higher consumer surplus as they induce firms to offer broader and overlapping product lines.

Keywords: second degree price discrimination, search, vertical differentiation, retail competition.

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1 Introduction

Since the classical work of Chamberlin (1933), a well known principle in economics is that firms differentiate their products in order to relax competition. Champsaur and Rochet (1989) (CR, thereafter) formalized this Chamberlinian incentive in a model in which quality choices are followed by price competition.[1] They showed that firms choose non-overlapping product lines because the incentives to soften price competition dominate over the incentives to discriminate consumers. Yet, in many markets, competing firms often carry overlapping qualities, even when this creates fierce competition among them. How can this fact be reconciled with the theory?

When consumers are not perfectly informed about firms' prices and qualities, they cannot choose their preferred option unless they incur search costs to learn and compare all options. Since the seminal work of Diamond (1971), the search literature has shown that the introduction of search frictions can have substantial effects on competition, no matter how search is modeled.[2] However, unlike CR, this literature has broadly neglected the possibility that firms engage in price discrimination through quality choices.[3] By combining these two literatures, this paper seeks to understand the interaction between search frictions and price discrimination in shaping the qualities and prices offered by competing firms.

By introducing search costs *à la* Varian (1980) into CR's model, we show that an arbitrarily small amount of search frictions is all it takes to rule out CR's equilibrium.[4] Intuitively, the firm carrying low qualities would now find it worthwhile to also carry high qualities in order to better discriminate the non-shoppers (who do not search) and attract some high-valuation shoppers. This type of deviation is not profitable in CR because, in the absence of search frictions, the rents on the overlapping qualities would

[1]Shaked and Sutton (1982) formalized the same idea in a model similar to Champsaur and Rochet's (1989), with the difference that firms are allowed to offer one quality only. Thus, in Shaked and Sutton (1982), there is no possibility to discriminate consumers at the firm level.

[2]Search models can essentially be classified as models of either simultaneous search (Burdett and Judd, 1983) or sequential search (Stahl, 1989). De los Santos *et al.* (2012) test which of the two processes best represents actual search for online books, and conclude in favor of the simultaneous search model, which is the approach we adopt in this paper.

[3]Unlike the current paper, in which we model second-degree price discrimination, Fabra and Reguant (2017) allow for third-degree price discrimination in markets with search costs.

[4]The same result would arise if we introduced search costs *à la* Diamond, i.e., if we assumed that all consumers have equal and positive search costs. However, as it is well known, this approach gives rise to the Diamond's paradox by which all firms behave as monopolists and consumers do not search. Therefore, this model would not be well-suited to analyze the interaction between competition and price discrimination: firms would not actually compete. The Varian's approach avoids this paradox, giving rise to comparative statics that replicate empirical findings regarding search behavior and price patterns.

be competed away.

In contrast, we show that search frictions give rise to an equilibrium with fully overlapping qualities, even when such equilibrium results in low profits.[5] When search is costly, the marginal incentives faced by firms mimic those of a monopolist: firms' incentives to discriminate consumers through quality choices dominate over their incentives to soften price competition. This induces firms to at least offer the quality range that allows them to implement the monopoly solution.

The comparative statics of equilibrium outcomes with respect to search frictions can be biased if quality choices are taken as given. Essentially, search frictions affect quality choices (i.e., whether product lines overlap or not), and through that, they end up affecting prices, qualities and consumer surplus. There are two effects at play: on the one hand, an arbitrarily small amount of search frictions intensifies competition and increases product variety by giving rise to overlapping quality choices; on the other, further increases in search frictions relax competition, eventually leading to prices above those in frictionless markets. In sum, while an increase in search frictions is in general anti-competitive, search frictions might also lead to lower prices and/or higher product variety, thus making consumers better off.[6]

Last, in building the equilibrium with non-overlapping qualities, we generalize CR to settings in which there exist consumers with low reservation prices (CR implicitly assume that even the lowest type has a sufficiently high reservation price). The solution gives rise to new equilibrium patterns, even if CR's qualitative prediction –namely, that in the absence of search costs, firms can credibly relax competition by carrying non-overlapping qualities– remains unchanged.

Our paper is related to two strands of the literature: (i) papers that analyze competition with search costs, and (ii) papers that characterize quality choices under imperfect competition.[7] The vast part of the search literature assumes that consumers search for

[5]This conclusion remains valid regardless of whether the *non-shoppers* visit one firm at random, or whether they visit the one that gives them higher ex-ante utility.

[6]In general, search costs are thought to relax competition, thus leading to higher prices, although not as intensively as the Diamond paradox would have anticipated (Diamond, 1971). There are some exceptions to this general prediction. Some recent papers have shown that search costs can lead to lower prices, particularly so when search costs affect the types of consumers who search. For instance, see Moraga-González *et al.* (2017) and Fabra and Reguant (2017).

[7]There is also a large empirical literature investigating price discrimination in markets where search costs matter, with a focus on price patterns. There are studies on gasoline markets, where consumers have the choice of paying for full-service or self-service gasoline at the same station, or of searching for competing stations (Shepard, 1991); the airline industry, where travellers can choose whether to fly in business or in economy class, or just in economy class but with certain restrictions (Borenstein and Rose, 1994; Gerardi and Shapiro, 2009); coffee shops (McManus, 2000), cereals (Nevo and Wolfram, 2002), theaters (Leslie, 2004), Yellow Pages advertising (Busse and Rysman, 2005), and cable TV (Crawford

one unit of an homogenous good, with two exceptions. Some search models allow for product differentiation across firms but, unlike ours, assume that each firm carries a single product.[8] Other search models allow firms to carry several products but, unlike ours, typically assume that consumers search for more than one ('multi-product search').[9] In these models, consumers differ in their preference for buying all goods in the same store ('one-stop shopping') rather than on their preferences for quality.[10] These differences are relevant. In the first type of search models, the single-product firm assumption leaves no scope for price discrimination within the firm. Hence, pricing is solely driven by competitive forces. In the second type of search models, the multi-product search assumption implies that discrimination is based on heterogeneity in consumers' shopping costs, which become the main determinant of firms' product choices (Klemperer, 1992).

Within the 'multi-product search' literature, two papers deserve special attention. In line with our results, Zhou (2014) finds that multi-product firms tend to charge lower prices than single-product firms. This is not driven by the interaction between competition and price discrimination, as in our paper, but rather by a 'joint search' effect, i.e., multi-product firms charge less because they gain more by discouraging consumers from searching competitors. In Rhodes and Zhou (2019), increases in search costs imply that consumers value one-stop shopping more, thus making it more likely that the equilibrium involves multi-product firms. For small search costs, Rhodes and Zhou (2019) predict asymmetric market structures with single-product and multi-product firms coexisting. The driving force underlying our predictions is quite different: since in our model consumers buy a single good, the multi-product firm equilibrium is not driven by one-stop shopping considerations but rather by firms' incentives to price discriminate consumers with heterogenous quality preferences. Despite these differences, our paper has one common prediction with both Rhodes (2014) and Rhodes and Zhou (2019): namely, search frictions can give rise to lower prices through their effect on endogenous product choices.

and Shum, 2007), among others.

[8]For models with horizontal product differentiation, see for instance Anderson and Renault (1999) and Bar-Isaac *et al.* (2012); see Ershow (2017) for an empirical application. Wildenbeest (2011) allows for vertical differentiated products but, unlike us, assumes that all consumers have the same preference for quality; hence, there is no scope for price discrimination. He finds that all firms use the same symmetric mixed strategy in utility space, which means that firms use asymmetric price distributions depending on the quality of their product. In contrast, we find that firms might use different pricing strategies for the same product, with this asymmetry arising because of price discrimination within the store.

[9]There is a recent strand of papers in the ordered search literature that analyze obfuscation by multi-product firms (Gamp, 2016; Petrikaite, 2017). Their emphasis is on the monopoly case. See Armstrong (2016) for a discussion.

[10]One-stop shopping considerations are also the driving force behind the evidence of price dispersion across stores documented by Kaplan *et al.* (2016).

As far as we are aware of, Garret *et al.* (2019) is the only paper that, like ours, introduces frictions in a model of price discrimination.[11] Given our focus on whether search frictions affect firms' ability to commit to asymmetric product lines, we build our analysis on a two-stage game –first, firms choose which qualities to carry and then, they decide how to price those qualities–, while Garret *et al.* (2019) rely on a one-stage game in which firms are not constrained in the qualities they can offer. Our model shows that the subgame they consider is indeed on the equilibrium path.

Last, our paper also relates to the literature that analyzes quality choices followed by imperfect competition, either quantity competition (Gal-Or, 1983; Wernerfelt, 1986; Johnson and Myatt 2003, 2006 and 2015) or price competition with horizontal differentiation (Gilbert and Matutes, 1993; Stole, 1995). As already noted by CR (p. 535), one of the main consequences of less competitive pricing is to induce wider and, very likely, overlapping product lines. While one may view search frictions as equivalent to other forms of imperfect competition, they are not. In models of imperfect competition, for the equilibrium with overlapping (i.e., symmetric) quality choices to exist, competition has to be sufficiently weak, e.g. as shown by Gal-Or (1983), under Cournot competition, the number of firms has to be sufficiently small. The same insight also applies to models of price competition with horizontal product differentiation. If there is little (horizontal) product differentiation, the equilibrium with overlapping product choices breaks down because the rents lost when dropping a low quality good are small as compared to the increase in profits from softening competition. In contrast, the impacts of search frictions on product choices are different as, even with arbitrarily small search frictions, firms do not have incentives to deviate from the equilibrium with overlapping product lines.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 reviews the competitive and monopoly solutions as these will be used in the rest of the analysis. Section 4 revisits CR's non-overlapping equilibrium in the absence of search frictions and shows that such an equilibrium no longer exists as soon as search frictions are introduced. Section 5 characterizes equilibria with full quality overlap and shows that these equilibria always exist. Section 6 discusses the robustness of the model to several extensions and Section 7 concludes. Proofs are postponed to the Appendix.

[11] Another set of related papers analyze pricing for add-ons. Ellison (2005) and Verboven (1999) consider models in which consumers are well informed about base product prices but don't know the price of the add-ons, unless they search. Critically, in these models the customers that are more likely to buy the add-ons are also less likely to search. Our model is not a model of add-on pricing because shoppers observe all prices and non-shoppers only those of the store they visit, and this applies symmetrically for both products regardless of their quality. Furthermore, our results hold regardless of whether there is correlation or not between consumers' quality preferences and search cost types.

2 Model Description

Consider a market served by two firms that carry a set of qualities Q_i in $\mathbb{R}^+$, $i = 1, 2$. A firm's product line Q_i can include qualities within an interval, or within a finite number of disjoint intervals.[12] The cost of a particular quality $q \in Q_i$, denoted $C(q)$, is assumed to be strictly increasing and convex, with $C(0) = C'(0) = 0$. There is a unit mass of consumers who buy at most one good. Consumers differ in their preference for quality, as captured by their type θ. Types are drawn from a continuous distribution $F(\theta)$ with density $f(\theta) > 0$ in a closed interval $\left[0, \bar{\theta}\right]$.[13] Following Mussa and Rosen (1978) (MR, thereafter), the utility of type θ buying quality q at a price $p(q)$ is given by

$$U(\theta) = \theta q - p(q),$$

while the utility of not buying a good is normalized to zero.

For tractability purposes, we provide closed form solutions for MR's and CR's leading specification, which has quadratic costs, $C(q) = q^2/2$, and uniformly distributed types in $\left[0, \bar{\theta}\right]$ (*quadratic-uniform* case).

We consider a two-stage game with the following timing. First, simultaneously and independently, firms choose their product lines Q_i, $i = 1, 2$. Once chosen, (Q_1, Q_2) become observable to firms but not to consumers. Second, firms post menus of contracts with different quality-price combinations, under the constraint that all the qualities offered by firm $i = 1, 2$ must be contained in its product line Q_i. Last, consumers choose which firm(s) to visit so as to learn their prices and product lines. In order to maximize their utility, consumers decide which quality to buy (if any) and from which firm among the one(s) they have visited. Following Varian (1980), we assume that there is a fraction $\mu \leq 1$ of consumers who always visit the two firms (the *shoppers*), and hence know where to find the lowest price for each quality. Since the remaining $1 - \mu$ consumers only visit one firm (the *non-shoppers*),[14] they can compare the prices for the qualities sold *within* one firm, but not *across* firms. We assume that the non-shoppers visit one of the two firms with equal probability.[15] In what follows, we use the fraction of non-shoppers $1 - \mu$

[12] In CR, product lines are constrained to be an interval. We will consider equilibria in which product lines are an interval too, but unlike CR, we will consider potential deviations outside the interval.

[13] Setting $\underline{\theta} = 0$ reduces the number of cases we need to consider, while at the same time it gives rise to new results that do not appear in CR. Even though they do not mention it explicitly, their results apply only to the case in which $\underline{\theta}$ is sufficiently large. This explains why some of the results we derive in Section 4 do not fully coincide with those in CR, even though the qualitative nature of the two remains unchanged.

[14] An implicit assumption is that the fraction μ and the distribution of types are uncorrelated. As we discuss in Section 6, our main results do not change if we allow for correlation between both.

[15] In some settings it may be reasonable to assume that non-shoppers observe product lines but not

as a proxy for search frictions. Accordingly, search frictions are lower the higher μ, with $\mu = 1$ representing a frictionless market.[16]

3 Perfect Competition and Monopoly Reviewed

For future reference, it is useful to review the solutions under perfect competition and monopoly.

At the competitive solution, the consumer's marginal utility equals marginal cost at his optimal quality choice, $\theta = C'(q)$. In the quadratic-uniform setting, quality at the competitive menu is $q_c(\theta) = \theta$, and since consumers extract all the surplus, $U_c(\theta) = \theta^2/2$.

The monopolist chooses a set of menus $\{q(\theta), p(\theta)\}$ to maximize profits

$$\pi = \int [p(\theta) - C(q(\theta))] f(\theta) \, d\theta,$$

subject to the incentive compatibility constraints, $U(\theta) = \theta q(\theta) - p(\theta) \geq \theta q(\theta') - p(\theta')$ for all $\{\theta, \theta'\}$; and participation constraints, $U(\theta) \geq 0$ for all θ.

Using the Envelope Theorem, the optimality condition $U'(\theta) = q(\theta)$ implies that each type must obtain utility

$$U(\theta) = U(\theta^*) + \int_{\theta^*}^{\theta} q(s) \, ds, \tag{1}$$

where θ^* is the lowest type being served. Since it is optimal to set $U(\theta^*) = 0$, prices can then be written as

$$p(\theta) = \theta q(\theta) - U(\theta) = \theta q(\theta) - \int_{\theta^*}^{\theta} q(s) \, ds.$$

Solving the monopoly problem, the optimal quality for type θ is characterized by

$$C'(q_m(\theta)) = \theta - \frac{1 - F(\theta)}{f(\theta)},$$

and the lowest type being served θ^* by

$$C(q_m(\theta^*)) = \left[\theta^* - \frac{1 - F(\theta^*)}{f(\theta^*)}\right] q_m(\theta^*).$$

As is well known, there is a downward distortion of quality for all types except for the highest one, and not all types are served.

their prices. Accordingly, we have also considered the case in which non-shoppers visit the store that gives them higher expected utility (and split randomly between the two stores in case of symmetry). The main results of the paper are strengthened. See Section 6.

[16] Garret *et al.* (2019) introduce search frictions using a more general specification, which encompasses ours. A key property that is common in both specifications is that, with some positive probability, some consumers visit one firm only.

In the quadratic-uniform case, the quality schedule is given by $q_m(\theta) = 2\theta - \bar{\theta}$ if $\theta \in [\bar{\theta}/2, \bar{\theta}]$ and zero otherwise. Utilities are $U_m(\theta) = (\theta - \bar{\theta}/2)^2$ if $\theta \in [\bar{\theta}/2, \bar{\theta}]$ and zero otherwise. Last, monopoly profits are $\pi_m = \bar{\theta}^2/12$. This is the MR solution to which we will frequently refer throughout the paper.

4 Equilibrium with No Quality Overlap

Our point of departure is CR's equilibrium. They show that firms give up opportunities to price discriminate heterogeneous consumers in order to relax competition. In particular, firms avoid any quality overlap - which would lead to prices equal to marginal costs for such qualities - and further relax competition by leaving a gap between the two firms' product lines. In particular, in the absence of search frictions, CR's equilibrium takes the following form:[17]

Proposition 1 *Consider the quadratic-uniform case. If $\mu = 1$, the pair of product lines $Q_1 = [0, q_1^+]$ and $Q_2 = [\bar{\theta}, \infty)$, with $q_1^+ = q_m(q_1^+) < \bar{\theta}$, constitutes a subgame perfect Nash equilibrium (SPNE) of the two-stage game.*

Proof. See the Appendix. ■

In equilibrium, firm 1 and 2 offer a range of low and high quality products, respectively, with a gap in between, i.e., $q_1^+ < q_2^-$. Firm 1 discriminates consumers types up to $\theta \leq q_1^+$, from whom it obtains monopoly profits, and sells quality q_1^+ to consumers $\theta \in [q_1^+, \tilde{\theta}]$, where type $\tilde{\theta}$ is indifferent between buying q_1^+ at $p_1(q_1^+)$ and $q_2^- = \bar{\theta}$ at $p_1(q_2^-)$. Firm 2 sells a single quality $q_2^- = \bar{\theta}$ to the remaining consumers. Qualities above $\bar{\theta}$ are not bought in equilibrium, but they play a strategic role as they discourage firm 1 from offering those qualities (in the absence of search frictions, such overlap would lead to Bertrand pricing). Thus, consumers $\theta \leq \tilde{\theta}$ buy inefficiently low qualities from firm 1, except for $\theta = q_1^+$, while consumers $\theta \geq \tilde{\theta}$ buy inefficiently high qualities from firm 2, except for $\bar{\theta} = q_2^-$.

As explained by CR, firms do not want to expand their product lines: whereas this would allow firms to better discriminate consumers in the gap (i.e., those who buy either q_1^+ or q_2^-), it would also intensify competition among them, leading to lower prices for all consumer types. There is however an important distinction between our equilibrium and the one characterized in CR's Proposition 4. Unlike CR, the schedule offered in

[17] As mentioned, we depart from CR on an important assumption: whereas they assume that a "consumer always buys something", thus implicitly assuming an infinite reservation price, in our model the participation constraint need not be satisfied for all types. This key difference explains why the results in this Proposition differ from those in CR's Proposition 3, in which each firm produces a unique quality at the extremes.

equilibrium by firm 1 to consumers $\theta \leq q_1^+$ is not affected at all by firm 2's offer; it is a MR type of schedule, with no distortion at the top of its quality range, $q_m(q_1^+) = q_1^+$.

Several forces contribute to this result. As in CR, both firms have incentives to keep q_1^+ and q_2^- apart. Firm 2 does so, not only to soften competition for consumers in the gap, but also to reduce the outside option of consumers upon whom it exerts local monopoly power (those who consume qualities above q_2^-). In the quadratic-uniform setting, these two forces push firm 2 all the way up to $q_2^- = \bar{\theta}$. Similarly, when q_1^+ and q_2^- are close enough, firm 1 also wants to reduce q_1^+ to both soften competition in the gap as well as to reduce the outside option of consumers upon whom it exerts local monopoly power (those who consume qualities below q_1^+). However, when q_1^+ and q_2^- are sufficiently apart, firm 1's problem changes radically. In firm 2's problem, q_1^+ is always a relevant outside option for firm 2's customers as higher types are always tempted to buy lower quality goods. But this is not always the case in firm 1's problem as lower types are not tempted to buy higher quality goods when q_2^- is sufficiently apart from q_1^+.

In fact, if q_1^+ drops below the equilibrium level in Proposition 1 while q_2^- stays unchanged, firm 1 faces a MR's monopoly problem because q_2^- is no longer a relevant outside option for firm 1's consumers. As soon as this happens, exercising full monopoly power upon these consumers dominates the gain from further softening competition for consumers in the gap. As a result, q_1^+ is not pushed further apart from q_2^-, but it instead remains at the corner where firm 1 can exercise maximum monopoly power upon its captive consumers. This is in contrast to CR's model, in which the implicit restriction that $\underline{\theta}$ cannot be too low stops q_1^+ from falling down enough so as to be sufficiently far from q_2^-. Therefore, the fundamental asymmetry in the incentives faced by the two firms never arises in CR. As a result, the price of q_1^+ in CR is determined *as if* each firm offered a single quality (see their Proposition 1), resulting in an equilibrium in which both firms offer a single quality at the two extremes of the quality range (see their Proposition 3). Despite this difference between the two models, the qualitative prediction remains the same: in the absence of search frictions, firms carry non-overlapping qualities in equilibrium.

However, the non-overlapping equilibrium is not robust to introducing search frictions, no matter how big or small. Intuitively, the presence of non-shoppers increases the incentives to price discriminate: not carrying the full product line stops firms from discriminating not only the shoppers in the gap, but also a wider range of non-shoppers whose preferred qualities are not carried by the firm. In turn, the presence of non-shoppers reduces the incentives to compete: the demands faced by firms become less elastic as price reductions do not attract non-shoppers. However, this reasoning would seem to suggest that the mass of non-shoppers needs to be large enough for these effects to be strong enough. Yet, and in contrast to other type of market imperfections, an arbitrarily small amount of search frictions is enough to rule out the equilibrium in Proposition 1.

Consider the equilibrium in Proposition 1.[18] In the presence of search frictions, by offering $Q_2 = [\bar{\theta}, \infty)$, firm 2 no longer prevents firm 1 from offering qualities above $\bar{\theta}$. Indeed, firm 1 can extract more rents from some of the non-shopper high types by offering them a higher quality, as opposed to selling them q_1^+.[19] Firm 1 might be discouraged from doing so if such a deviation intensified competition for the shoppers it serves in equilibrium, potentially leading to lower profits overall. However, as we show in the proof of the next proposition, it is always possible to find a sufficiently high quality that firm 1 would find it profitable to sell to non-shoppers without attracting any shoppers. While such a deviation may be enough to rule out the equilibrium in Proposition 1, there is a more profitable deviation for firm 1, which is to offer high qualities not only to extract more rents from non-shopper high types, but also to attract some shopper high types. Although this deviation also intensifies the competition for shoppers in the gap, this effect is of second order compared to the increase in the profits made out of the high types. It is as if the gap $q_1^+ < q_2^- = \bar{\theta}$ acted as a buffer. In sum, the non-overlapping equilibrium characterized in Proposition 1 does not survive the introduction of non-shoppers, no matter how few they are.[20]

Proposition 2 *Consider the quadratic-uniform case. If $\mu < 1$, the non-overlapping equilibrium characterized in Proposition 1 does not exist.*

Proof. See the Appendix. ■

5 Equilibrium with Full Quality Overlap

In this section we analyze symmetric equilibria with full quality overlap. We proceed by backwards induction by first analyzing the second stage (the choice of quality-price

[18] To be sure, the deviation that rules out the existence of the equilibrium in Proposition 1 would also rule out CR's equilibrium under their implicit assumption of $\underline{\theta}$ high enough. Indeed, deviation profits would be even higher because under CR's equilibrium firm 1 is even more constrained to extract rents from the non-shoppers.

[19] Note that the equilibrium characterized in Proposition 1 would remain if we were to restrict firm 1 to deviations within the gap, i.e., $q \in (q_1^+, q_2^-]$; for instance, if we adopted the ad-hoc restriction that Q_i has to be an interval.

[20] Understanding the equilibrium implications of firm 1's deviation is out of the scope of the analysis. For instance, we do not know if an asymmetric equilibrium with non-overlap for low qualities and overlap for high qualities exists. Our aim is simply to demonstrate that the equilibrium in which both firms choose qualities in an interval with a gap in between no longer exists in the presence of non-shoppers.

menus for given product lines) and then the first stage (the choice of product lines).

Consider the choice of quality-price menus. Standard Bertrand arguments imply that there cannot exist an equilibrium in pure strategies: competition for the shoppers would induce firms to slightly undercut prices for all qualities, while rent extraction from the non-shoppers would discourage them from setting prices that are too low. The non-existence of pure strategy equilibria is shared by most models of simultaneous search, starting with Varian (1980).

In Varian's model, firms randomize over a single variable: the price at which they offer an homogeneous product. Since the firm that charges the lowest price attracts all shoppers, it is relatively simple to characterize firms' profits. Matters are more difficult when firms' strategies involve randomizing over quality-price menus. In particular, it might not always be possible to rank the menus offered by the two firms: whereas some consumer types might prefer to buy from firm 1, others might prefer to buy from firm 2 (this occurs when the menus are *non-ordered*). For this reason, and following Garret *et al.* (2019), we restrict attention to equilibria with *ordered* menus, defined as follows:

Definition 1 *(Ordered menus) Consider two menus* $\{q(\theta), p(\theta)\}$ *and* $\{\hat{q}(\theta), \hat{p}(\theta)\}$*, giving utilities* $U(\theta)$ *and* $\hat{U}(\theta)$*, with* $U(\theta') \geq \hat{U}(\theta')$ *for some* $\theta' \in \left[0, \bar{\theta}\right]$*. These two menus are ordered if* $U(\theta) \geq \hat{U}(\theta)$ *for all* $\theta \in \left[0, \bar{\theta}\right]$*, with strict inequality whenever* $U(\theta) > 0$*. In this case, menu* $\{q(\theta), p(\theta)\}$ *is said to be more generous than menu* $\{\hat{q}(\theta), \hat{p}(\theta)\}$*.*

The menus that firms offer in an ordered-menu equilibrium can be indexed by their generosity, which we denote by x, $\{p_x(\theta), q_x(\theta)\}$. Hence, in equilibrium firms can be thought of as choosing generosity $x \in [\underline{x}, \overline{x}]$ according to a distribution $G(x)$, where $\underline{x}$ and $\overline{x}$ respectively denote the generosity of the *least* and *most* generous menus in the support.

With ordered menus, if firm i chooses a menu of generosity x, it attracts all shoppers (regardless of their valuation) if the rival chooses a less generous menu, an event that occurs with probability $G(x)$. Hence, when a firm chooses a menu with generosity $x \in [\underline{x}, \overline{x}]$, its expected equilibrium profits can be written as

$$\Pi_x = \left(\frac{1-\mu}{2} + G(x)\mu\right)\pi_x, \tag{2}$$

where π_x are the per-consumer expected profits,

$$\begin{aligned}\pi_x &= \int_0^{\bar{\theta}} \left[p_x(\theta) - C(q_x(\theta))\right] f(\theta)\, d\theta. \\ &= \int_0^{\bar{\theta}} \left[\theta q_x(\theta) - U_x(\theta) - C(q_x(\theta))\right] f(\theta)\, d\theta.\end{aligned}$$

To characterize the mixed strategy equilibrium, we start by assuming that the initial quality range does not constrain firms' offers, i.e., $Q_i = [0, \infty)$, $i = 1, 2$. In a symmetric equilibrium with ordered menus, when a firm offers the least generous menu, the rival firm is offering more generous menus with probability one, i.e., $1 - G(\underline{x}) = 1$. Hence, the firm only serves its fraction $(1 - \mu)/2$ of the non-shoppers. Since profits are thus proportional to monopoly profits, firms simply face a monopoly problem when choosing $\underline{x}$. It follows that the optimal least generous menu coincides with the monopoly solution. Since all menus in the support of a mixed strategy equilibrium generate the same expected profits, at any symmetric ordered-menu equilibrium, expected profits are equal to monopoly profits over the non-shoppers.

Proposition 3 *Assume $Q_i = [0, \infty)$, $i = 1, 2$. For all $\mu \in (0, 1)$, at any symmetric equilibrium with ordered menus, the least generous menu is given by the MR solution. Hence, expected equilibrium profits for each firm are $\pi_m(1 - \mu)/2$.*

As the share of shoppers goes up, expected equilibrium profits go down from the monopoly solution (when almost all consumers are non-shoppers, $\mu \to 0$) to zero (when almost all consumers are shoppers, $\mu \to 1$).

The proposition above also implies that at the most generous menu that is offered in equilibrium, expected profits must be equal to the monopoly profits from serving the non-shoppers. Since when a firm offers the most generous menu, the rival firm is offering less generous menus with probability one, i.e., $G(\overline{x}) = 1$, this implies

$$\Pi_{\overline{x}} = \left(\frac{1 - \mu}{2} + \mu\right) \pi_{\overline{x}} = \frac{1 - \mu}{2} \pi_m,$$

or equivalently,

$$\pi_{\overline{x}} = \frac{1 - \mu}{1 + \mu} \pi_m.$$

At the most generous menu a firm makes lower per-consumer expected profits than at the least generous menu, but it is more likely to serve more customers. Since per-consumer profits π_x are decreasing in generosity, and the right hand side of the above equation is decreasing in μ, it follows that $\overline{x}$ must be increasing in μ, i.e., the more shoppers there are, the more generous is the most generous menu that is offered in equilibrium.

Interestingly, μ affects the most generous menu, but not the least one (which remains as in the MR solution). Hence, in markets with higher μ (i.e., lower search costs) there is more dispersion in the set of menus offered in equilibrium.

The equilibrium characterization is completed by computing the distribution function that firms use to choose the generosity of their menus.

Proposition 4 *For all* $\mu \in (0,1)$ *, in a symmetric equilibrium with ordered menus, firms choose generosity* $x \in [\underline{x}, \overline{x}]$ *according to*

$$G(x) = \frac{1-\mu}{2\mu}\left(\frac{\pi_m}{\pi_x} - 1\right).$$

Proof. It simply follows from equating equation (2) to equilibrium expected profits, equal to the MR's profits of serving the non-shoppers. ■

As μ goes up, more mass is put on more generous menus. In the limit, as $\mu \to 1$, almost all the mass is put at the lower bound, resulting in a menu which is arbitrarily close to the competitive solution.

Last, in order to show that an equilibrium with the above properties indeed exists, we rely on Garret *et al.* (2019).[21]

Proposition 5 *Assume* $Q_i = [0, \infty)$ *,* $i = 1, 2$ *and consider the quadratic-uniform case. There exists a symmetric equilibrium with ordered menus.*

Proof. See Theorem S1 in the online appendix of Garret *et al.* (2019). ■

At the equilibrium proposed by Garret *et al.* (2019), the highest type always obtains the efficient quality, and its price smoothly goes down under more generous menus. The quality of all the other types is distorted downwards, but quality distortions decrease as firms offer more generous menus. The range of types that are served in equilibrium is enlarged under more generous menus. Indeed, there is a one-to-one mapping between the menu's generosity and the lowest type that is served under the most generous menu.

Garret *et al.* (2019) prove existence, but not uniqueness. They conjecture that their equilibrium is the unique smooth ordered equilibrium of the game, but this does not rule out the possibility that there might exist other equilibria with ordered menus that are not smooth (e.g., with bunching). However, even if this was the case, expected equilibriums profits would remain the same (Proposition 3), which is all that matters for our current purposes; namely, to show existence of an equilibrium with overlapping product lines.

So far, we have restricted attention to unconstrained qualities $Q_i = [0, \infty)$. However, the same analysis would go through for narrower ranges, as long as firms are not constrained from offering the MR qualities. More binding quality ranges would however lead to lower profits as they would constrain firms from extracting monopoly profits out of the non-shoppers.

[21] Garret *et al.* (2019) consider the two-types case. Their online appendix also contains the equilibrium characterization for the continuum type model in the quadratic-uniform case.

Lemma 1 *Assume* $Q_i = [0, q^+]$ *with* $q^+ \geq q_c(\bar{\theta}) = q_m(\bar{\theta})$, $i = 1, 2$. *For all* $\mu \in (0, 1)$, *at any symmetric equilibrium with ordered menus, expected equilibrium profits for each firm are* $\pi_m(1 - \mu)/2$. *All other symmetric quality intervals yield strictly lower expected equilibrium profits.*

Proof. See the Appendix. ■

We are now ready to analyze first stage quality choices. The next proposition characterizes quality choices at the (symmetric) SPNE.

Proposition 6 *For all* μ, *there exists a SPNE equilibrium with overlapping product lines* $Q_i = [0, q^+]$ *for* $i = 1, 2$, *with* $q^+ \geq q_c(\bar{\theta}) = q_m(\bar{\theta})$.

Proof. See the Appendix. ■

Deviations from symmetric product lines are unprofitable whenever they constrain firms from implementing the MR solution. If the deviant carried fewer qualities than at the MR solution, the deviant would not be able to obtain monopoly profits over the non-shoppers, making such a deviation unprofitable. Furthermore, there cannot exist symmetric equilibria with narrower quality ranges as firms would find it optimal to enlarge their first stage product lines until they no longer constrain their second stage choices. The intuition is simple: a firm that deviated would be able to at least obtain monopoly profits over the non-shoppers, regardless of the menus offered by the rival firm.

To conclude, in the presence of search frictions, no matter how big or small, the symmetric SPNE involves overlapping quality choices that do not constrain firms from implementing the monopoly solution over the non-shoppers. The mass of shoppers determines how close the equilibrium is to the monopoly solution ($\mu = 0$) or to the competitive solution ($\mu \to 1$). In all cases, the equilibrium involves overlapping quality choices over the full range.

Comparing the non-overlapping and overlapping equilibria Search frictions affect outcomes through two channels: they impact price and quality when the overlapping equilibrium prevails (as already discussed in Garret *et al.* (2019)), and they impact product lines when the equilibrium switches from the non-overlapping to the overlapping type when arbitrarily small search frictions are introduced (Proposition 6).

Due to the latter effect, the pattern of expected prices for given qualities depicts a discontinuity when search frictions are arbitrarily small. Indeed, at the non-overlapping equilibrium firms are able to sustain prices that are strictly above marginal costs for all qualities on sale. In contrast, at the overlapping equilibrium with arbitrarily small search frictions, all qualities are offered at marginal cost with probability close to one. Using the terminology of Armstrong (2015), non-shoppers create a positive search externality to the shoppers as these end up paying lower prices while the range of qualities on offer is enlarged. However, as search frictions become more important, higher prices are played with greater probability, eventually leading to prices that are higher than under the non-overlapping equilibrium with no search costs. Hence, the conventional wisdom that search frictions lead to higher prices applies in this model, but only when search frictions do not change equilibrium product lines, i.e., everywhere except in the limit when search frictions become arbitrarily small.

The range of qualities that are actually bought under the non-overlapping equilibrium is much narrower than at the overlapping equilibrium because the preferred qualities of those consumers in the "gap" are not available. Hence, introducing an arbitrarily small amount of search costs also implies a discontinuous jump in the range of qualities bought.

Last, putting the price and quality impacts together, consumers are better off with mild search frictions than in frictionless markets: prices are lower and there is more product variety. However, when search frictions are sufficiently high, consumers are faced with a trade-off as prices are higher than in the absence of search frictions but there is more product variety. Accordingly, there exists a level of search frictions above which consumers are worse off than in frictionless markets, and vice-versa.

6 Extensions and Variations

In the preceding sections we characterized quality and price choices under three assumptions which we now seek to relax: (i) duopoly; (ii) search cannot be conditioned on product line choices (as these were assumed non-observable prior to search); and (iii) consumers' search frictions and quality preferences are uncorrelated. Our focus is on the existence of the "overlapping" equilibrium.

N **symmetric firms oligopoly** A similar logic as in the duopoly case also allows to conclude that all firms carrying all qualities constitutes a SPNE for all $\mu < 1$. In the second stage, the least generous menu in the mixed strategy equilibrium is given by the monopoly solution given that at this menu the firm only sells to the non-shoppers (since firms are symmetric, there cannot be a mass at the least generous menu). Thus, equilibrium profits are a fraction $(1-\mu)/N$ of monopoly profits just as in the duopoly case. Alternatively, if one firm deviates by dropping one or more qualities, the deviant (weakly) reduces its profits as it will not be able to implement the monopoly solution. Hence, the presence of shoppers restores the monopolist's incentives to carry all qualities just as in the duopoly case.

Observable product choices and directed search by the non-shoppers In the main model we assumed that consumers do not observe product lines prior to visiting the stores. In particular, we assumed that the non-shoppers visit one of the two stores with equal probability, regardless of their product choices. Instead, suppose now that non-shoppers visit the store that gives them higher expected utility, given firms' (observable) product choices and expected prices.[22] Allowing search to be conditioned on product choices would strengthen our main result: when directed search is allowed, offering more product variety would allow firms to not only better discriminate, but also to attract more non-shoppers.

Directed search by the non-shoppers only affects pricing when firms have chosen asymmetric product lines (with symmetric product lines, expected prices are also symmetric so it is irrelevant whether search is directed or random). We have formally studied this problem elsewhere (Fabra and Montero, 2017), but in a simpler setting of two types of consumers and two qualities (high and low). Suppose that one firm carries the low quality version and the other carries both versions. The first observation is that directed search requires expected prices for the low quality to be equal across stores, so in equilibrium non-shopper low types are indifferent as to which store to visit. Second, all non-shopper high types visit the multi-product firm not only because expected prices for the low quality version are equal across stores but also because incentive compatibility makes them indifferent between the two qualities.

The directed search outcome just described differs from the one when non-shoppers split evenly between the two stores (i.e., when product lines are not observable). In this latter case, the multi-product firm charges lower prices for the low quality good. This more aggressive pricing is explained by the multi-quality firm's ability to segment non-shoppers. Now, to rebalance firms' pricing incentives from this latter case to the case of directed search, more than half of the non-shopper low types must visit the multi-product store until their expected prices converge, ultimately reducing the single-product firm's market share and profit. This implies that the incentives to deviate from the case in which both firms carry both qualities to carry just the low quality one are necessarily weaker under directed search. A similar reasoning applies to a deviation to carry just the high quality version.

We see no reason to expect that this reasoning would fail in our more general setting of many qualities and consumer types, but it would be complex to prove existence of equilibrium. Thus, we believe that our main conclusion –namely, that the "overlapping" equilibrium is robust for all $\mu < 1$– remains valid regardless of whether product lines are

[22] This interpretation of non-shoppers as sophisticated buyers is closer to that in the clearing-house model *à la* Baye and Morgan (2001).

observable (and there is directed search by the non-shoppers) or not.

Correlation between search frictions and quality preferences Last, we have so far assumed that shoppers and non-shoppers are equally likely to be either high or low types. However, this may not hold in practice. For instance, if low types are lower income consumers with more time to search, then the non-shoppers are more likely to be high types.[23] Alternatively, if high types enjoy shopping for their preferred (high quality) product, then non-shoppers are more likely to be low types. Ultimately, this is an empirical question whose answer may vary depending on the type of product or context considered. However, as far as the predictions of the model are concerned, it is inconsequential whether the correlation between search frictions and quality preferences is positive, negative or non-existent.[24]

To formalize this, one can assume that the fraction of shoppers might vary across consumer types, i.e., $\mu(\theta)$ represents the fraction shoppers of type θ, with $\mu = \int \mu(\theta) f(\theta) d\theta$. If $\mu(\theta)$ decreases in θ, there is positive correlation between search frictions and quality types as the high types are less likely to be shoppers (i.e., higher types have higher search costs). The analysis of product and price choices without search frictions remains intact since all consumers are shoppers by definition. As for the analysis with search frictions, expected profits at a symmetric equilibrium with ordered menus remain proportional to monopoly profits, thus implying that the incentive structure remains unchanged. As such, the "overlapping" equilibrium always exists just as in the case with no correlation between search and quality preferences.

7 Conclusions

In this paper we have analyzed a model of quality choice followed by competition with price-quality menus in markets with search frictions. We have found that an arbitrarily small amount of search frictions is enough to overturn the prediction that firms are always able to soften competition by carrying non-overlapping product lines, as in the seminal

[23] This negative correlation would be consistent with the evidence in Kaplan and Menzio (2016), as employed workers have less time to search for low prices than unemployed workers.

[24] One may also argue that being a shopper should be an equilibrium decision; for example, if consumers, after learning about their types, had the option to pay some "information-acquisition" cost to become shoppers. We believe that our main results would prevail if this information-acquisition cost were distributed over the population of consumers such that, regardless of their type, some fraction of consumers decided to remain uninformed in equilibrium. If, for some reason, all high types decided to become informed in equilibrium, the schedules in the overlapping equilibrium may be distorted downwards, i.e., the least generous schedule may no longer be the monopoly schedule but something slightly more competitive (i.e., more generous).

paper of Champsaur and Rochet (1989). Instead, a small amount of search frictions create head-to-head competition by inducing firms to carry overlapping product lines.

We have shown that, through product choice, search frictions have important implications for market outcomes beyond their well studied price effects. Furthermore, we have shown that analyzing the price effects of search frictions without endogenizing product lines can sometimes lead to overestimating the anticompetitive effects of search frictions.

The multi-product nature of firms also adds important twists to the analysis of competition in the presence of search frictions. An important departure from Varian (1980) is that goods within a store cannot be priced independently from each other. In particular, the incentives to segment consumer types imply that firms' offers have to satisfy the incentive compatibility constraints. In the same spirit of Varian (1980), we have also shown that search frictions give rise to dispersion in menus, i.e., in the prices for each quality as well as in the qualities actually bought for each consumer.

Admittedly, there are several motives other than the ones studied in this paper that shape firms' product choices. In particular, throughout the analysis we have assumed that firms do not incur any fixed cost of carrying a product. This modelling choice was meant to highlight the strategic motives underlying product choice. However, fixed costs of carrying a product (which could arguably be higher for high quality products),[25] could induce firms to offer fewer and possibly non-overlapping products. Our prediction is not that competitors should always carry overlapping product lines. Rather, our analysis suggests that if their product lines do not overlap in markets in which search frictions matter, it must be for reasons other than firms' attempts to soften competition through product choice- for instance, due to the presence of fixed costs.

To the extent that firms could collude to coordinate their product choices (as reported by Sullivan (2016) in the context of the super-premium ice cream market),[26] competition authorities should remain vigilant if competitors' product lines do not overlap - particularly so in markets in which fixed costs (at the product level) are not relevant but

[25] In some cases, such costs can be substantial, e.g. firms have to advertise that they are carrying an additional product, or the transaction costs of dealing with an additional provider can sometimes be high. The marketing literature has analyzed several factors explaining the limited number of products sold per firm. For instance, Villas-Boas (2004) analyzes product line decisions when firms face costs of communicating about the different products they carry to their customers. They show that costly advertising can induce firms to carry fewer products as well as to charge lower prices for their high-quality goods.

[26] See also the *NY Times* note quoted in the paper. Although it is difficult to divide "smooth" and "chunky" flavors in low and high-quality options, the logic of our result may apply as well. The ice-cream company that focuses on chunky flavors may need to reprice its existing offer downwards if it decides to also carry smooth flavors and compete head-to-head on these flavors with the rival company just carrying them. But this is profitable as long as exist some fraction of smooth non-shoppers.

consumers find it costly to search.

Appendix: Proofs

Proof of Proposition 1 Suppose that in the first stage firms choose $Q_1 = [0, q_1^+]$ and $Q_2 = [q_2^-, \bar{\theta}]$, with $q_1^+ \leq q_2^-$, respectively. As long as q_1^+ and q_2^- are not too far apart (in a way to be made precise shortly), equilibrium prices $p_1^+ \equiv p_1(q_1^+)$ and $p_2^- \equiv p_2(q_2^-)$ at the second stage can be obtained from solving the auxiliary pricing game where firms carry a single quality (see CR's Proposition 1),

$$p_1^+ = \arg\max_{p_1} (p_1 - C(q_1^+)) \left(F(\tilde{\theta}) - F(\theta') \right) \tag{3}$$

and

$$p_2^- = \arg\max_{p_2} (p_2 - C(q_2^-)) \left(1 - F(\tilde{\theta}) \right), \tag{4}$$

where $\tilde{\theta} = (p_2 - p_1)/(q_2^- - q_1^+)$ is the shopper indifferent between options (q_1^+, p_1^+) and (q_2^-, p_2^-) and $\theta' = p_1/q_1^+$ is the last shopper buying from firm 1. Solving the system of first-order conditions yields

$$p_1^+ = C(q_1^+) + (F(\tilde{\theta}) - F(\theta')) \frac{q_1^+(q_2^- - q_1^+)}{f(\tilde{\theta})(q_2^- - q_1^+) + f(\theta')q_1^+} \tag{5}$$

and

$$p_2^- = C(q_2^-) + (1 - F(\tilde{\theta})) \frac{(q_2^- - q_1^+)}{f(\tilde{\theta})}. \tag{6}$$

In addition to these two prices, we need equilibrium schedules for qualities other than $q_1^+$and q_2^-. Firm 2's schedule corresponds to the Mussa-Rosen schedule reviewed in Section 3, the only difference being that $U(\theta_2^*)$, the utility of the lowest type ($\theta_2^* \geq \tilde{\theta}$) served under the schedule, is no longer zero but equal to $U(\theta_2^*) = \theta_2^* q_2^- - p_2^-$ (firm 1's presence has increased the low end outside option of consumers buying from firm 2). So, using (1), firm 2's price schedule can be written as

$$p_2(\theta) = \theta q_2(\theta) - U(\theta) = \theta q_2(\theta) - \int_{\theta_2^*}^{\theta} q_2(s)\, ds - U(\theta_2^*),$$

which is then used to obtain firm 2's optimal quality schedule,

$$C'(q_2(\theta)) = \theta - \frac{F(\bar{\theta}) - F(\theta)}{f(\theta)}.$$

For the quadratic-uniform setting, this schedule reduces to $q_2(\theta) = q_m(\theta) = 2\theta - \bar{\theta}$ for all $\theta \in [\theta_2^*, \bar{\theta}]$ and zero otherwise.

We still need a expression for θ_2^* as a function of first-stage variables. Using the "smooth-pasting" condition $q_2(\theta_2^*) = q_2^-$, we obtain $\theta_2^* = (q_2^- + \bar{\theta})/2$. From here, and using (5) and (6), we can express firm 2's (first-stage) payoff as a function of q_1^+ and q_2^- as follows

$$\Pi_2(q_1^+, q_2^-) = \int_{\theta_2^*}^{\bar{\theta}} \left[\left(\theta - \frac{F(\bar{\theta}) - F(\theta)}{f(\theta)} \right) q_2(\theta) - C(q_2(\theta)) \right] f(\theta)\, d\theta$$
$$- U(\theta_2^*) \left(F(\bar{\theta}) - F(\theta_2^*) \right) + \left(p_2^- - C(q_2^-) \right) \left[F(\theta_2^*) - F(\tilde{\theta}) \right].$$

Firm 1's price schedule $p_1(\theta)$ is more involved because there are two outside options to be handled. While low types still have to be prevented from buying the low quality (outside) option (for a payoff that has been normalized to zero), high types now have to be prevented from buying a high quality option offered by firm 2, in particular q_2^-. Formally, there is a type $\hat{\theta} \leq \theta_1^{**} \leq \tilde{\theta}$, where θ_1^{**} is the highest-type consumer buying from firm 1 under the price schedule $p_1(\theta)$, such that, by incentive compatibility, the utility of types $\theta < \hat{\theta}_1$ is still given by (1), while the utility of types $\theta > \hat{\theta}_1$ now takes the form (preventing lower types from mimicking higher types)

$$U(\theta) = U(\theta_1^{**}) - \int_{\theta}^{\theta_1^{**}} q_1(s)\, ds$$

for $\theta \in [\hat{\theta}_1, \theta_1^{**}]$. Combining this latter with (1) for $\theta \in [\theta_1^*, \hat{\theta}_1]$, where θ_1^* is the lowest type being served, we proceed as before to obtain firm 1's optimal quality schedule

$$C'(q_1(\theta)) = \theta - \frac{F(\hat{\theta}_1) - F(\theta)}{f(\theta)}.$$

For the quadratic-uniform setting, this reduces to $q_1(\theta) = 2\theta - \hat{\theta}_1$ for all $\theta \in [\theta_1^*, \theta_1^{**}]$. Only type $\hat{\theta}_1$ obtains the efficient quality. Types $\theta > \hat{\theta}_1$ are offered inefficiently high qualities, while types $\theta < \hat{\theta}_1$ are offered inefficiently low qualities.

In addition to the smooth-pasting condition $q_1(\theta_1^{**}) = 2\theta_1^{**} - \hat{\theta}_1 = q_1^+$ and $U(\theta_1^{**}) = \theta_1^{**} q_1^+ - p_1^+$, we also know from $U(\theta_1^*) = 0$ that $q_1(\theta_1^*) = 2\theta_1^* - \hat{\theta}_1 = 0$ and $U(\theta_1^{**}) = \int_{\theta_1^*}^{\theta_1^{**}} q_2(\theta) f(\theta) d\theta$. From here, we can express firm 1's (first-stage) payoff as a function of q_1^+ and q_2^- as follows

$$\Pi_1(q_1^+, q_2^-) = \int_{\theta_1^*}^{\theta_1^{**}} \left[\left(\theta - \frac{F(\hat{\theta}_1) - F(\theta)}{f(\theta)} \right) q_1(\theta) - C(q_1(\theta)) \right] f(\theta)\, d\theta$$
$$- U(\theta_1^{**}) \left(F(\theta_1^{**}) - F(\hat{\theta}_1) \right) + \left[p_1^+ - C(q_1^+) \right] \left(F(\tilde{\theta}) - F(\theta_1^{**}) \right).$$

Moving backwards to the first stage to solve the system $q_1^+ = \arg\max_{q_1} \Pi_1(q_1, q_2^-)$ and $q_2^- = \arg\max_{q_2} \Pi_2(q_1^+, q_2)$, we quickly arrive at the corner $q_1^+ = q_1(\theta_1^{**}) = \theta_1^{**} = \hat{\theta}_1 =$

$(2-\sqrt{2})\bar{\theta}$ and $q_2^- = \bar{\theta}$, where $\partial\Pi_1(q_1^+, q_2^-)/\partial q_1 < 0$ and $\partial\Pi_2(q_1^+, q_2^-)/\partial q_2 > 0$. The reason q_1^+ cannot be reduced any further despite $\partial\Pi_1(q_1^+, q_2^-)/\partial q_1 < 0$ is because when $q_1^+ < (2-\sqrt{2})\bar{\theta}$ (while holding q_2^- fixed at $\bar{\theta}$), p_1^+ is no longer governed by the system (3) and (4), but by a MR schedule with no distortion at the top of its quality range, i.e., $q_1(q_1^+) = q_m(q_1^+) = q_1^+$ and $p_1(q_1^+) = q_1^+ q_1(q_1^+) - \int_{\theta_1^*}^{q_1^+} q_1(s)\, ds = 3(q_1^+)^2/4$ (recall that from $U(\theta_1^*) = 0$, we have $\theta_1^* = q_1^+/2$). We say that at this point q_1^+ is too far apart from q_2^-, so that the schedule $q_1(\theta)$ offered in equilibrium by firm 1 to consumers $\theta \leq q_1^+$ is not affected at all by firm 2's offer (and CR's Proposition 1 no longer applies).[27]

When firm 1's schedule is governed by a MR schedule, firm 1's payoff $\Pi_1(q_1^+, q_2^-)$ needs to be modified accordingly: $\theta_1^{**} = \hat{\theta}_1 = q_1^+$, $\theta_1^* = q_1^+/2$, $q_1(\theta) = 2\theta - q_1^+$ and $p_1^+ = 3(q_1^+)^2/4$. Firm 2's payoff also needs to be modified slightly in that p_2^- is no longer given by (6) but needs to be obtained directly from (4). Solving the system $q_1^+ = \arg\max_{q_1} \Pi_1(q_1, q_2^-)$ and $q_2^- = \arg\max_{q_2} \Pi_2(q_1^+, q_2)$ for these new payoff functions, we quickly arrive at the same corner: $q_1^+ = (2-\sqrt{2})\bar{\theta}$ and $q_2^- = \bar{\theta}$, where $\partial\Pi_1(q_1^+, q_2^-)/\partial q_1 > 0$ and $\partial\Pi_2(q_1^+, q_2^-)/\partial q_2 > 0$. The reason q_1^+ cannot be increased any further despite $\partial\Pi_1(q_1^+, q_2^-)/\partial q_1 > 0$ is because when $q_1^+ > (2-\sqrt{2})\bar{\theta}$ (while holding q_2^- fixed at $\bar{\theta}$), p_1^+ is again governed by the system (3) and (4).[28]

The proof concludes calling CR's Proposition 5: if the pair of quality ranges $Q_1 = [0, q_1^+ = (2-\sqrt{2})\bar{\theta}]$ and $Q_2 = [q_2^- = \bar{\theta}, \bar{\theta}]$ constitutes a SPNE of the quality game, then the pair of quality ranges $Q_1 = [0, q_1^+ = (2-\sqrt{2})\bar{\theta}]$ and $Q_2' = [q_2^- = \bar{\theta}, +\infty)$ also constitutes a SPNE of the quality game, leading to the exact same pricing schedules and payoffs.

Proof of Proposition 2 The proof is divided in two steps. The first step consists in showing that for $\mu \to 1$ it is profitable for firm 1 to deviate to carry some $q > \bar{\theta}$ to better discriminate non-shopper high types without attracting any additional shopper (for simplicity, we restrict attention to deviations to a single quality q). Because $\mu \to 1$, prices p_1^+ and p_2^- in Proposition 1 remain unchanged had firm 1 not deviated to carry some quality $q > \bar{\theta}$. According to Proposition 1, these prices can be obtained from (5) and (6), that for the quadratic-uniform setting reduces to $p_1^+ = 3(q_1^+)^2/4$ and $p_2^- = q_1^+\bar{\theta}$, with $q_1^+ = (2-\sqrt{2})\bar{\theta}$.

Now, for quality $q > \bar{\theta}$ to be profitable and feasible to be offered to non-shopper

[27] Note that when q_1^- is sufficiently low, i.e., when $q_1^- < 0.53\bar{\theta}$ in our quadratic-uniform setting (while holding q_2^- fixed at $\bar{\theta}$), prices are again governed by the system (3) and (4) because from now on firm 1 finds it optimal to offer a single quality, q_1^+, and give up on any segmentation upon lower types. Segmenting lower types, which are not that valuable anymore, would only introduce (lower) qualities that would force firm 1 to leave more rents with higher types.

[28] Note that at the equilibrium, firms' payoffs can still be decomposed as in CR's Proposition 4 because their Proposition 1 pricing equilibrium is just valid.

high types for some price p (while holding p_1^+ fixed), the following profitability and participation conditions must hold, respectively

$$p - C(q) > p_1^+ - C(q_1^+)$$

and

$$\bar{\theta}q - p > \bar{\theta}q_1^+ - p_1^+$$

which lead to

$$\bar{\theta}q - C(q) > \bar{\theta}q_1^+ - C(q_1^+)$$

that for the quadratic-setting reduces to $q < 2\bar{\theta} - q_1^+ = \sqrt{2}\bar{\theta}$. So, if firm 1 deviates to carry $q = \sqrt{2}\bar{\theta}$, it can barely attract the non-shopper highest type $\bar{\theta}$ for a price of $p_1^+ + C(q) - C(q_1^+) = p_1^+ + \bar{\theta}(q - q_1^+)$, reporting no extra profit on non-shoppers (and obviously nothing extra on shoppers).

Similarly, for quality $q > \bar{\theta}$ to be profitable and feasible to be offered to shopper high types for some price p (while holding firm 2's price offers fixed) the following two conditions must hold, respectively

$$p - C(q) > 0$$

and

$$\bar{\theta}q - p > \bar{\theta}^2 - p_2^-$$

which lead to

$$\bar{\theta}q - C(q) > \bar{\theta}^2 - p_2^-$$

that for the quadratic-setting (recall that $p_2^- = \bar{\theta}q_1^+$) reduces to $q < \bar{\theta} + \sqrt{2q_1^+\bar{\theta} - \bar{\theta}^2} = \sqrt{2}\bar{\theta}$. So again, if firm 1 deviates to carry $q = \sqrt{2}\bar{\theta}$, it can barely attract the shopper highest type $\bar{\theta}$ for a price of $p = C(q)$, reporting no extra profit on shoppers but a strict loss on non-shoppers equal to

$$\frac{1-\mu}{2}(p_1^+ - q_1^+)(1 - F(\theta')) > 0$$

where $\theta' = (C(q) - p_1^+)/(q - q_1^+) < \bar{\theta}$ is the non-shopper that is just indifferent between taking option (q_1^+, p_1^+) and option $(q, C(q))$.

Since deviating to carrying $q = \sqrt{2}\bar{\theta}$ reports no extra profit to firm 1 when aiming this quality at non-shoppers, by pricing it at $p = p_1^+ + C(q) - C(q_1^+)$, and a loss when aiming it at both shoppers and non-shoppers, by pricing it at $p = C(q)$, from a standard continuity argument there exists a quality $q^{NS} \in (\bar{\theta}, \sqrt{2}\bar{\theta})$ that leaves firm 1 just indifferent between aiming q^{NS} exclusively at non-shoppers, by pricing it at $p' \in (p_1^+ + C(q^{NS}) - C(q_1^+), p_1^+ +$

$\bar{\theta}(q^{NS} - q_1^+))$, and aiming it at both shoppers and non-shoppers, by pricing it at $p'' \in (C(q^{NS}), \bar{\theta}q^{NS} + p_2^- - \bar{\theta}^2) < p'$.[29]

Having established that carrying q^{NS} is a profitable deviation for $\mu \to 1$, the second step of the proof is to show that firm 1 wants to deviate even further, to carry $q < q^{NS}$ to attract some shoppers high type (along with non-shoppers high type) with positive probability. When firm 1 deviates to carry $q < q^{NS}$, she anticipates two competitive responses from firm 2 in the pricing stage. The first is that firm 2 will price q more aggressively now. In the absence of non-shoppers, this (equilibrium) response would be to price q at cost $C(q)$. In the presence non-shoppers, however, the price competition for selling q will be in mixed strategies. The corresponding equilibrium is straightforward to characterize: Firm 1 will choose price $p_1(q) \in \{[\underline{p}(q), \bar{p}(q)], p^u(q)\}$ according to some cumulative distribution function $H_1(p; q)$, with $\underline{p} > C(q)$, $p^u > \bar{p}$ and $H_1(\bar{p}; q) < 1$ (i.e., firm 1 will put a mass $1 - H_1(\bar{p}; q) > 0$ at the upper bound p^u, where it only serves non-shoppers), while firm 2 will choose price $p_2(q) \in [\underline{p}(q), \bar{p}(q)]$ according to some (atomless) function $H_2(p; q)$.

The second response of firm 2 is a consequence of the first. Since firm 2 expects to loose some shoppers high type to firm 1 with positive probability, and hence, have fewer inframarginal shoppers buying q_2^-, firm 2 will also respond to $q < q^{NS}$ by lowering the price of q_2^- from its equilibrium level in Proposition 1.[30] So, in deciding whether to deviate to $q < q^{NS}$, firm 1 must trade off (i) the benefit of attracting some shoppers high type while extracting more from non-shoppers high type against (ii) the cost of increasing competition for shoppers in the gap. But at the margin, when $q = q^{NS}$, the latter effect is zero, because $H_1(\bar{p}; q^{NS}) = 0$ and $\underline{p}(q^{NS}) = \bar{p}(q^{NS})$.[31] Hence, the optimal deviation necessarily entails $q < q^{NS}$, where the marginal benefit of increasing q is equal to its marginal cost.

Proof of Lemma 1 Equilibrium profits at a symmetric equilibrium are fully determined by profits at the least generous menu, which is given by the monopoly solution. Hence, as long as the first stage quality range does not constrain firms from implementing it, i.e., for $Q_i = [0, q^+]$ for $i = 1, 2$, with $q^+ \geq q_m(\bar{\theta}) = q_c(\bar{\theta})$, expected equilibrium prof-

[29] For example, $q^{NS} = 1.4\bar{\theta}$ for $\mu = 0.995$.

[30] Note that p_2^- also becomes random since it is decided simultaneosly with $p_2(q)$, but within a much tighter interval. We do not need to make any of this explicit for our proof.

[31] Firm 1's cost $\ell(q)$ of setting $q < q^{NS}$ can be expressed as

$$\ell(q) \sim \int_{\underline{p}(q)}^{\bar{p}(q)} \Delta(p; q) H_1(p; q) dH_2(p; q)$$

where $\Delta(p; q)$ is firm 2's inframarginal (high-type) shoppers lost to firm 1 when $p_1(q) < p_2(q)$. It follows that $\ell'(q^{NS}) = 0$.

its remain unchanged. For narrower product lines, it still holds that at any symmetric equilibrium with ordered menus expected profits are given by the highest profits that can be made out of the non–shoppers. However, these must be strictly below monopoly profits as firms do not carry all the qualities needed to extract monopoly profits from the non-shoppers.

Proof of Proposition 6 Consider deviations from a candidate symmetric equilibrium with $Q = [0, q^+]$, with $q^+ \geq q_m(\bar{\theta}) = q_c(\bar{\theta})$ to $Q'_i = [q_i^-, q_i^+]$ with $q_i^- \geq 0$ and/or $q_i^+ \leq q^+$. Clearly, equilibrium profits would remain the same if the firm enlarges its product line, as it would not be constrained to implement the monopoly solution over the non-shoppers. Consider thus deviations to $q_i^- > 0$ and/or $q_i^+ \leq q^+$. At the least generous menu, firm i would be constrained to implement the optimal scheme over the non-shoppers. Hence, its profits must go down relative to the candidate equilibrium unless firm j plays a mass point at the least generous menu. However, this leads to a contradiction, as firm j would then be making lower profits than at the monopoly solution, which it can guarantee to its self by offering the MR solution.

References

[1] Armstrong, M. (2015), Search and Rip-offs Externalities, *Review of Industrial Organization* 47(3), 273–302.

[2] Armstrong, M. (2016), Ordered Consumer Search, CEPR Discussion Papers 11566.

[3] Anderson, S. and Renault, R. (1999), Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model, *RAND Journal of Economics* 30(4), 719-735.

[4] Bar-Isaac, H., Caruana, G. and Cuñat, V. (2012), Search, Design, and Market Structure, *American Economic Review* 102(2), 1140-60.

[5] Baye, M. and J. Morgan (2001), Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets, *American Economic Review* 91(3), 454-474.

[6] Borenstein, S. and Rose, N.L. (1994), Competition and Price Dispersion in the U.S. Airline Industry, *Journal of Political Economy* 102(4), 653-68.

[7] Brander, J. and Eaton, J. (1984), Product-line Rivalry, *American Economic Review* 74, 323-334.

[8] Burdett, K. and Judd, K.L. (1983), Equilibrium Price Dispersion, *Econometrica* 51(4), 955-969.

[9] Busse, M. and Rysman, M. (2005), Competition and Price Discrimination in Yellow Pages Advertising, *RAND Journal of Economics* 36(2), 378-390.

[10] Chamberlin, E. H. (1933), The Theory of Monopolistic Competition. Cambridge, MA: Harvard University Press.

[11] Champsaur, P. and Rochet, J.-C. (1989), Multi-product Duopolists, *Econometrica* 57, 533-557.

[12] Chen, L., Mislove, A. and Wilson, C. (2016), An Empirical Analysis of Algorithmic Pricing on Amazon Marketplace, mimeo.

[13] Crawford, G. and Shum, M. (2007), Monopoly Quality Degradation in Cable Television, *Journal of Law and Economics* 50, 181-219.

[14] Diamond, P. (1971), A Model of Price Adjustment, *Journal of Economic Theory* 3(2), 156-168.

[15] De Los Santos, B., Hortaçsu, A. and Wildenbeest, M. (2012), Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior, *American Economic Review* 102(6), 2955-80.

[16] Ellison, G. (2005), A Model of Add-on Pricing, *Quarterly Journal of Economics* 120(2), 585-637.

[17] Ershow, D. (2016), The Effect of Consumer Search Costs on Entry and Quality in the Mobile App Market, mimeo, University of Toronto.

[18] Fabra, N. and Montero, J.-P. (2017), Product Choice and Price Discrimination in Markets with Search Costs, CEPR Discussion Paper 12046.

[19] Fabra, N. and Reguant, M. (2018), A Model of Search with Price Discrimination, CEPR working paper.

[20] Gal-Or, E. (1983), Quality and Quantity Competition, *Bell Journal of Economics* 14(2), 590-600.

[21] Gamp, (2017), Guided Search, mimeo.

[22] Garret, D., Gomes, R., and Maestri, L. (2019), Competitive Screening under Heterogeneous Information, *Review of Economic Studies*, forthcoming.

[23] Gerardi, K. and Shapiro, A. (2009), Does Competition Reduce Price Dispersion? New Evidence from the Airline Industry, *Journal of Political Economy* 117(1), 1-37.

[24] Gilbert, R. and Matutes, C. (1993), Product Line Rivalry with Brand Differentiation, The Journal of Industrial Economics 41(3), 223-240.

[25] Hong, H., and Shum, M. (2006), Using Price Distributions to Estimate Search Costs, *RAND Journal of Economics* 37(2), 257-75.

[26] Iyer, G., and Seetharaman, P.B. (2003), To Price Discriminate or Not: Product Choice and the Selection Bias Problem, *Quantitative Marketing and Economics* 1, 155-178.

[27] Johnson, J.P., and Myatt, D.P. (2003), Multi-product Quality Competition: Fighting Brands and Product Line Pruning, *American Economic Review* 93(3), 748-774.

[28] Johnson, J.P., and Myatt, D.P. (2006), Multi-product Cournot Oligopoly, *RAND Journal of Economics* 37(3), 583-601.

[29] Johnson, J.P., and Myatt, D.P. (2015), The Properties of Product Line Prices, *International Journal of Industrial Organization* 43, 182-188.

[30] Kaplan, G., Menzio, G., Rudanko, L. and Thachter, N. (2016), Relative Price Dispersion: Evidence and Theory, NBER Working Paper No. 21931.

[31] Kaplan, G., and G. Menzio (2016), Shopping Externalities and Self-Fulfilling Unemployment Fluctuations, *Journal of Political Economy* 124(3), 771-825.

[32] Klemperer, P. (1992), Equilibrium Product Lines: Competing Head-to-Head May be Less Competitive, *American Economic Review* 82(4), 740-55.

[33] Leslie, P. (2004), Price Discrimination in Broadway Theater, *RAND Journal of Economics* 35(3), 520-541.

[34] Mc Manus, B. (2007), Nonlinear Pricing in an Oligopoly Market: The Case of Specialty Coffee, *RAND Journal of Economics* 38(2), 513-533.

[35] Moraga González, J.L., Sandor, Z. and Wildenbeest, M. (2017), Prices and Heterogeneous Search Costs, *RAND Journal of Economics* 48(1), 125-146.

[36] Mussa, M. and Rosen, S. (1978), Monopoly and Product Quality, *Journal of Economic Theory* 18, 301-317.

[37] Nevo, A. and Wolfram, C. (2002), Why Do Manufacturers Issue Coupons? An Empirical Analysis of Breakfast Cereals?, *RAND Journal of Economics* 33, 319-339.

[38] Petrikaite, V. (2017), Consumer Obfuscation by a multi-product Firm, mimeo.

[39] Rhodes, A. (2015), multi-product Retailing, *Review of Economic Studies* 82(1), 360-390.

[40] Rhodes, A., and Zhou, J. (2019), Consumer Search and Retail Market Structure, Management Science, *forthcoming*.

[41] Romero, J. (2015), The Effect of Hysteresis on Equilibrium Selection in Coordination Games, *Journal of Economic Behavior and Organization* 111, 88-105.

[42] Seim, K. and Sinkinson, M. (2016), Mixed Pricing in Online Marketplaces, *Quantitative Marketing and Economics* 14(2), 129-155.

[43] Shaked, A. and Sutton, J. (1982), Relaxing Price Competition Through Product Differentiation, *Review of Economic Studies* 49(1), 3-13.

[44] Shepard, A. (1991), Price Discrimination and Retail Configuration, *Journal of Political Economy* 99(1), 30-53.

[45] Stahl, D.O. (1989), Oligopolistic Pricing with Sequential Consumer Search, *American Economic Review* 79(4), 700-712.

[46] Stole, L. (1995), Nonlinear Pricing and Oligopoly, *Journal of Economics & Management Strategy* 4, 529-562.

[47] Stole, L. (2007), Price Discrimination and Competition, *Handbook of Industrial Organization*, 3 (eds. M. Armstrong and R. Porter), North-Holland.

[48] Sullivan, C. (2016), The Ice Cream Split: Empirically Distinguishing Price and Product Space Collusion, mimeo, University of Michigan.

[49] Tirole, J. (1988), The Theory of Industrial Organization, Cambridge, MA: MIT Press.

[50] Verboven, F. (1999), Product Line Rivalry and Market Segmentation - with an Application to Automobile Optional Engine Pricing, *Journal of Industrial Economics* 47(4), 399-425.

[51] Villas-Boas, M. (2004), Communication Strategies and Product Line Design, *Marketing Science* 23, 304-316.

[52] Varian, H. (1980), A Model of Sales, *American Economic Review* 70(4), 651-659.

[53] Vives, X. (2011), Strategic Supply Function Competition With Private Information, *Econometrica* 79 (6).

[54] Wernerfelt, B. (1986), Product Line Rivalry: Note, *American Economic Review* 76(4), 842-44.

[55] Wernerfelt, B. (1988), Umbrella Branding as a Signal of New Product Quality: An Example of Signalling by Posting a Bond, *RAND Journal of Economics* 19(3), 458-466.

[56] Wildenbeest, M. R. (2011), An Empirical Model of Search with Vertically Differentiated Products, *RAND Journal of Economics* 42(4), 729-757.

[57] Zhou, J. (2014), Multi-product Search and the Joint Search Effect, *American Economic Review* 104(9), 2918-2939.