Auctions with Unknown Capacities: Understanding Competition among Renewables*

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Abstract

The energy transition will imply a change in the competitive paradigm of electricity markets. Competition-wise, one distinguishing feature of renewables versus fossil-fuels is that their marginal costs are known but their available capacities are uncertain. Accordingly, in order to understand competition among renewables, we analyze a uniform-price auction in which bidders are privately informed about their capacities. Due to capacity uncertainty, renewables partially mitigate market power as compared to conventional technologies. In particular, firms exercise market power by either withholding output when realized capacities are large, or by raising their bids above marginal costs when realized capacities are small. Since markups are decreasing in realized capacities, a positive capacity shock implies that firms offer to supply more at reduced prices, giving rise to lower but also more volatile market prices. An increase in capacity investment depresses market prices, which converge towards marginal cost only when total installed capacity is sufficiently large, or when the market structure is sufficiently fragmented.

Keywords: multi-unit auctions, electricity markets, renewables, forecasts.

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1 Introduction

Ambitious environmental targets, together with decreasing investment costs, have fostered the rapid deployment of renewable energy around the world. Installed renewable capacity has more than doubled over the last ten years and it is expected to further increase during the coming decade. In several jurisdictions, the goal to achieve a carbon-free power sector by 2050 will require an almost complete switch towards renewable energy sources.[1] This global trend begs the question: how will electricity markets perform in the future once renewables become the major energy source?

Whereas competition among conventional fossil-fuel generators is by now well understood (e.g. Borenstein, 2002; von der Fehr and Harbord, 1993; Green and Newbery, 1992, among others) much less is known about competition among wind and solar producers (which we broadly refer to as *renewables*). Competition-wise, there are two key differences between conventional and renewable technologies. First, the marginal cost of conventional plants depends on their efficiency rate as well as on the price at which they buy the fossil fuel. In contrast, the marginal cost of renewable generation is essentially zero, as plants produce electricity out of freely available natural resources (e.g. wind or sun). Second, the capacity of conventional power plants is well known, as they tend to be available at all times (absent rare outages). In contrast, the availability of renewable plants is uncertain, as it depends on weather conditions that are difficult to forecast (Gowrisankaran et al., 2016).[2] Hence, the move from fossil fuel generation towards renewable sources will imply a change in the competitive paradigm. Whereas the previous literature has analyzed environments in which marginal costs are private information but production capacities are publicly known (Holmberg and Wolak, 2018; Vives, 2011), the relevant setting will soon become one in which marginal costs are known (and essentially zero) but firms' available capacities are private information.

In this paper, we build a model that captures this new competitive paradigm, which

[1] The International Renewable Energy Agency estimates that compliance with the 2017 Paris Climate Agreement will require overall investments in renewables to increase by 76% in 2030, relative to 2014 levels. Europe expects that over two thirds of its electricity generation will come from renewable resources by 2030, with the goal of achieving a carbon-free power sector by 2050 (European Commission, 2012). Likewise, California has recently mandated that 100% of its electricity will come from clean energy sources by 2045.

[2] Strictly speaking, not all renewable power sources share these characteristics, and our analysis applies mainly to wind and solar power. Other renewable technologies, such as hydro electricity, are storable or have a production that can be managed very much like in the case of thermal plants (e.g. biomass plants).

we apply to electricity markets. We assume that firms' available capacities are subject to common and idiosyncratic shocks: the former are publicly observable, the latter are privately known. Firms compete by submitting a price-quantity pair (i.e., an inverted-L supply function), indicating the minimum price and the maximum quantity they are willing to supply. The auctioneer calls firms to produce in increasing price order until total demand is satisfied, and all accepted offers are paid at the market-clearing price (uniform-price auction).

We show that capacity uncertainty mitigates market power, but it does not fully eliminate it. In equilibrium, firms behave competitively only when the common shock is so large that all firms but one would have enough capacity to serve total demand, regardless of their idiosyncratic shocks. In all other cases, market prices add a positive markup to marginal cost. At the symmetric equilibrium, mark-ups are decreasing in the firm's realized capacity, i.e. the more capacity a firm has, the lower is the price at which it is willing to supply it. This reflects the standard trade-off faced by competing firms: decreasing the price leads to an output gain (quantity effect), but it also leads to a lower market price if the rival bids below (price effect). Since firms gain more from the quantity effect when their realized capacity is large, they are more eager to set lower mark-ups.

In the short run, positive capacity shocks induce firms to submit lower bids, leading to lower market prices. This gives rise to price dispersion, which is inherently linked to capacity volatility through the effects of strategic behaviour. In the long-run, an increase in total investment also depresses expected market prices, which converge towards marginal costs only when total installed capacity is sufficiently large (or when the market structure is sufficiently fragmented).

Other recent papers have also analyzed competition among renewables by introducing capacity uncertainty (Acemoglu et al., 2017 and Kakhbod et al., 2018). Unlike us, these papers assume Cournot competition, i.e., they constrain firms to exercise market power by withholding output. In this paper we allow firms to choose both prices and quantities, and show that in equilibrium firms exercise market power *à la* Cournot only when realized capacities exceed demand. Instead, for smaller capacities, firms exercise market power *à la* Bertrand-Edgeworth, i.e., by bidding all their capacity at a price above marginal cost. The focus of Acemoglu et al. (2017) and Kakhbod et al. (2018) is also different from ours. Acemoglu et al. (2017) focus on the effects of common ownership between conventional and renewable plants. They show that common ownership mitigates the price depressing effect of renewables as strategic firms withhold more output from their conventional plants when there is more renewable generation.[3] Kakhbod et al. (2018) focus on the heterogeneity in the availability of renewable sources across locations and show that firms withhold more output when their plants are closely located, i.e., when their output is highly and positively correlated.

Holmberg and Wolak (2018) and Vives (2011) analyze auctions in which firms are privately informed about their marginal costs rather than about their capacities. In Holmberg and Wolak (2018) firms compete by choosing price offers, while in Vives (2011) firms compete by choosing continuously differentiable supply functions. In Holmberg and Wolak (2018), a lower (higher) cost realization shifts firms' price offers down (up) while firms' quantity offers remain unchanged.[4] This is unlike our model in which a higher (lower) capacity realization shifts the supply function downwards and outwards (upwards and inwards) as both prices and quantities respond to private information. In Vives (2011) firms receive an imprecise signal about their costs, which are correlated across firms. Since the market price aggregates information, firms submit steeper supply functions so as to produce less (more) when market prices are high (low), which is when marginal costs are likely to be high (low) as well. Hence, due to cost correlation, private information in Vives (2011) gives rise to less competitive equilibria than under full information. Instead, in the private values model, the equilibrium of Vives (2011) converges to its full information counterpart.

Interestingly, the comparison of our paper with Holmberg and Wolak (2018) and Vives (2011) uncovers a fundamental difference between introducing private information on costs or on capacities. Private information has two potential effects on bidding behaviour: through the bidders' price offers and through the quantities they sell. In a private-values setting, when costs are private information, a firm's realized cost determines its bid (higher-cost firms offer higher prices), but it does not affect (for a given bid) the quantity

[3] This effect is not present in our model since, in order to focus on the strategic interaction among renewables, we assume either that conventional power plants are not present, or that they are owned by independent producers, thus constraining renewable producers from raising prices above the conventional plants' marginal costs. If we allowed the independent conventional producers to exercise market power, an increase in renewables would reduce the residual demand faced by the conventional producers, thus reducing their incentives to increase prices. However, under common ownership, their incentives to exercise market power would be reinforced as renewables would be part of their inframarginal output.

[4] More precisely, Holmberg and Wolak (2018) do not allow firms to specify their quantity offers, and any (unknown) changes in capacities are uncorrelated with cost shocks.

allocated to the rival. Instead, when capacities are private information, a firm's realized capacity not only determines its bid (larger firms offer lower prices) but it also affects the quantity allocated to the rival.[5] This effect makes the equilibrium price offers steeper, as a firm with a small realized capacity has stronger incentives to raise its bid than if quantities were not affected by the rivals' private information.[6] This difference also explains why, in contrast to Holmberg and Wolak (2018) and Vives (2011), the equilibrium in our model always departs from the one under full information. The properties of our equilibrium are closer to the ones with positively affiliated signals on costs, even if we assume that capacity shocks are independent across firms.[7]

The comparison of our model with the ones in Holmberg and Wolak (2018) and Vives (2011) also shows that, with private values, equilibrium market prices tend to be more elastic to capacity than to cost shocks. The channel is two-fold. First, as already argued, since equilibrium bids are steeper, changes in the available capacities imply large changes in the bids. And second, since equilibrium price offers are decreasing in realized capacities, firms offer lower prices precisely when they can produce more.

A natural question is whether and how information precision affects bidding behaviour and thus market outcomes. Lagerlöf (2016) also addresses this question, but he does so in the context of the Hansen-Spulber model of price competition under cost uncertainty (Hansen, 1988; Spulber, 1995). We reach a similar conclusion even if the channels differ: the more uncertainty firms face, the weaker is their market power. Capacity asymmetries induce firms to compete less fiercely because their optimal bids tend to be away from each other. However, firms need to be informed about their realized capacities in order to be aware of their asymmetries. Otherwise, if their expected capacities are equal, they bid symmetrically, resulting in head-to-head competition. As a consequence, the highest profit levels are obtained when capacities are publicly known, while the lowest profit levels are obtained under unknown capacities. In between, when firms are privately informed about their own capacities, equilibrium profits are higher the more precise is the information about the rivals' capacities. This suggests that firms might be better off

[5] The reason is that, contingent on being the high bidder, the rival serves the residual demand net of the firm's capacity.

[6] Note that, conditionally on being smaller than the rival, the expected residual demand faced by a firm is larger the smaller its own capacity.

[7] With positively affiliated signals, a higher cost signal increases (for a given price) the probability of selling a higher quantity as the rival's cost, and thus its price offer, is also expected to be higher. Therefore, a firm's quantity is affected by the rival's private information, just as in our model.

exchanging information on their available capacities in order to sustain higher equilibrium profits. Our results provide a theoretical explanation to the experimental findings in Hefti et al. (2019), which are reminiscent of the literature on Treasury auctions (LiCalzi and Pavan, 2005) showing that noise in the demand function rules out the seemingly collusive equilibria that arise otherwise (Back and Zender, 2001).

Finally, even though we have motivated our model in the context of electricity markets, our setup is applicable to other relevant contexts where auctions are used. The literature on this topic typically assumes that bidders are privately informed about their costs or valuations, but tends to omit that bidders may also be privately informed about the maximum number of units they are willing to buy or sell.[8] Yet, this source of private information is present in many auction settings other than electricity markets. In Treasury Bill auctions (Hortasu and McAdams, 2010), banks are privately informed about their hedging needs and, hence, about how many bonds they are willing to buy; in Central Banks' liquidity auctions (Klemperer, 2019), banks are privately informed about the volume of toxic assets they can provide as collateral; or in emission permits auctions, firms are privately informed about their carbon emissions and hence about the amount of permits they need to purchase, to name just a few.[9] Likewise, firms have private information on capacities in a wide range of markets that can be analyzed through the lens of auction theory (Klemperer, 2003), e.g. the markets for hotel bookings or ride-hailing services, in which firms are privately informed about the number of empty rooms or available cars. Our model provides a framework that can account for this source of private information.

The reminder of the paper is structured as follows. In Section 2 we describe the model and we interpret it in the context of electricity markets. Section 3 characterizes the bidding equilibria when firms' capacities are private information. In Section 4 we assess the impact of private information and information precision on equilibrium outcomes. Section 5 provides some extensions and variations of our main model, including issues related to the market structure (more than two firms, and ex-ante asymmetric firms)

[8]Arguably, incomplete information on capacities can be captured through incomplete information on costs or valuations. A capacity constraint can be modeled as an infinite cost or as a zero valuation beyond a certain number or units. However, this structure is not typically studied and papers either assume that bidders are not capacity constrained or do not allow for discontinuities in marginal costs.

[9]Other examples include spectrum auctions (Milgrom, 2004), procurement auctions for a wide range of goods and services, auctions for electricity generation capacity (Fabra, 2018; Llobet and Padilla, 2018) or auctions for investments in renewables Cantillon (2014), among others.

and to the market design (a ban on capacity withholding). Finally, Section 6 closes by discussing what we learn from the model to understand competition among renewables. All proofs are relegated to the appendix.

2 The Model

Two (ex-ante symmetric) firms $i = 1, 2$ compete in a market to serve a perfectly price-inelastic demand, denoted as $\theta > 0$. Firms can produce at a constant marginal cost $c \geq 0$ up to their available capacities,[10] which are assumed to be random. In particular, the available capacity of firm i, denoted $k_i > 0$, is the result of two additive components, $k_i = \beta\kappa + \varepsilon_i$. The parameter $\beta \in [0, 1]$ in the common component captures the proportion of each firm's nameplate capacity κ that is available. The idiosyncratic shock ε_i is distributed according to $\Phi(\varepsilon_i|\kappa)$ in an interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, with $E(\varepsilon_i) = 0$. As a result, firm i's available capacity k_i is distributed according to $G(k_i) = \Phi(k_i - \beta\kappa|\kappa)$ in the interval $k_i \in \left[\underline{k}, \overline{k}\right]$, where $\underline{k} = \beta\kappa + \underline{\varepsilon}$ and $\overline{k} = \beta\kappa + \overline{\varepsilon}$. We denote the density as $g(k_i)$ and we assume it is positive in the whole interval $\left[\underline{k}, \overline{k}\right]$. Note that firms' available capacities are positively correlated through the common component, but they are conditionally independent given β. Firm i can observe its own idiosyncratic shock but not that of its rival, i.e., available capacities are private information.

Firms compete on the basis of the bids submitted to an auctioneer. Each firm simultaneously and independently submits a price quantity pair (b_i, q_i), where b_i is the minimum price at which it is willing to supply the corresponding quantity q_i. We assume $b_i \in [0, P]$, where P denotes the "market reserve price",[11] and $q_i \in [0, k_i]$, for $i = 1, 2$.

The auctioneer ranks firms according to their price offers, and calls them to produce in increasing rank order. In particular, if firms submit different prices, the low-bidding firm is ranked first. If firms submit equal prices, firm i is ranked first with probability $\alpha(q_i, q_j)$ and it is ranked second with probability $1 - \alpha(q_i, q_j)$. We assume a symmetric

[10] We can set $c = 0$ without loss of generality. In the context of renewables, a positive c could reflect the operation and maintenance variable costs. It could also be interpreted as the opportunity cost of firms whenever they have the option of selling this output through other channels, e.g. bilateral contracts or balancing markets.

[11] This price can be interpreted as the marginal cost of the next available technology, as long as it is offered competitively, or as an explicit or implicit price cap. Explicit price caps are common in the day-ahead electricity market.

function $\alpha(q_i, q_j) = \alpha(q_j, q_i) \in (0,1)$.[12] If firm i is ranked first it produces $\min\{\theta, q_i\}$, while if it is ranked second it produces $\max\{0, \min\{\theta - q_j, q_i\}\}$.

Firms receive a uniform price per unit of output, which is set equal to the market clearing price. For $b_i \geq b_j$, the market clearing price is defined as

$$p = \begin{cases} b_i & \text{if } q_i > \theta, \\ b_j & \text{if } q_i \leq \theta \text{ and } q_i + q_j > \theta, \\ P & \text{otherwise.} \end{cases}$$

In words, if the quantity offered by the winning bid(s) exceeds total demand, the market clearing price is set by the highest accepted bid. Otherwise, the lowest non-accepted bid sets the market price (if no such bid exists because all the quantity offered has been accepted, then $p = P$).[13]

The profits made by each firm are computed as the product of their per unit profit margin $(p - c)$ and their dispatched output. As explained before, both the market price p as well as firms' outputs are a function of demand θ, the price (b_i, b_j), and the quantities (q_i, q_j) offered by both firms. Firms, which are assumed to be risk neutral, bid so as to maximize their individual expected profits, given their realized capacities.

2.1 Interpreting the Model

The model described above is applicable to a wide range of auction settings in which firms' capacities (or demands) have a private information component. As already discussed, in electricity auctions, the supply of renewable generators depends on their available capacities, which are subject to idiosyncratic shocks. In emissions permit auctions, the demand of permits by polluting firms depends on their emissions and abatement costs. In Treasury Bill auctions, the demand by banks depend on their hedging needs, which in turn depend on how many loans and deposits they have taken. Similarly, in Central Bank's auctions, the demand of liquidity depends on the amount and quality of

[12] Hence, when firms' quantity offers are equal, $\alpha(q, q) = 1/2$. We do not need to specify $\alpha(q_i, q_j)$ outside of the diagonal as it is inconsequential for equilibrium bidding.

[13] Assuming that the market price is set at the lowest non-accepted bid when the quantity offered by the winning bid(s) does not exceed total demand is made for analytical convenience, with no impact on equilibrium outcomes. In particular, it avoids situations where firms want to offer a quantity slightly below total demand in order to have the rival set the price at a higher level.

the banks' collateral.[14]

Our model has being built to understand the future performance of electricity markets, and so it captures some of their key characteristics. Notably, similarly to most electricity markets in practice, the model assumes that firms compete by submitting a finite number of price-quantity pairs to an auctioneer, who then allocates output and sets market prices according to such bids.[15]

Our model also reflects key properties of electricity demand and supply. Regarding demand, we have assumed that it is price-inelastic and known by the time firms submit their bids. This is justified on two grounds. First, electricity retail prices typically do not reflect movements in spot market prices.[16] And second, System Operators regularly publish very precise demand forecasts before the market opens. Extending the model to allow for some degree of demand elasticity and/or demand uncertainty would add a layer of complexity without significantly affecting its main predictions.

Regarding supply, conventional technologies are implicitly present in the model through the price cap P, which can be interpreted as the marginal costs of coal or gas plants (as long as they behave competitively). This reflects our aim of shedding light on the performance of electricity markets during the late stages of the energy transition, when conventional technologies will mainly serve as back-up. Our model could be easily extended to allow for strategic behaviour of the conventional technologies.

Importantly, one of the core assumptions of our analysis is that firms possess private information that allows them to perfectly forecast their available capacities. In practice, as captured by our model, the availability of renewable resources is subject to common and idiosyncratic shocks. Firms' available capacities are thus correlated through the com-

[14]Similar examples can be found in markets which are not organized as formal auctions: how much oil an oil producer is willing to sell depends on the remaining oil in the well; how many available cars a ride-hailing company has depends on how many drivers are on service, net of those who are already occupied; how many rooms a hotel is willing to offer online depends on how many rooms have been booked through other channels; how much cloud computing space a firm is willing to offer depends on how much excess capacity it has above its own data needs; or how much olive oil a firm is willing to sell depends on whether its harvest was good or bad.

[15]While in practice firms are allowed to submit more than one step in their bidding functions, because of tractability reasons we restrict them to submitting just one. The same applies to other papers in the electricity auctions literature (Holmberg and Wolak, 2018; Fabra et al., 2006). Analyzing the model with multiple steps is beyond the scope of this paper.

[16]Even where they do, consumers typically do not have strong incentives or the necessary information to optimally respond to changes in spot market prices. The empirical evidence shows that this is the case in the Spanish electricity market, the only country so far where Real Time Pricing has been implemented as the default option for all households (Fabra et al., 2019).

mon shock component, albeit imperfectly so due to the presence of idiosyncratic shocks. While System Operators typically publish forecasts of the common shock at the national or regional level, the idiosyncratic component remains each firm' private information. Indeed, through the monitoring stations installed at the renewable plants' sites, firms have access to local weather measurements that are not available to the competitors. Beyond weather conditions, the plants' availability might be subject to random outages and maintenance schedules that only their owners are aware of. Accordingly, in the presence of private information, each firm is better informed about its own available capacity than its competitors.

To quantify this claim empirically, we have collected data from the Spanish electricity market to perform and compare forecasts of each plant's production, with and without firms' private information. In particular, we have obtained proprietary data of six renewable plants corresponding to their hourly production and their own available capacity predictions at the time of bidding, for a period of two years. We have also gathered the capacity predictions computed by the Spanish System Operator (Red Eléctrica de España) and the one-day ahead predictions of the Spanish weather agency (AEMET) at the finer regional level that is available, close to the plant's location. We have used OLS to forecast each plants' hourly availability, with and without the firms' proprietary local data. Figure 1 plots the distribution of the forecast errors and Table 1 summarizes the mean and standard deviations of the corresponding forecast errors. The evidence suggests that firms' private information allows them to significantly improve the precision of the forecasts of their own plants' available capacities.[17] Interestingly, when the private forecast is used, the national forecast, while still statistically significant, has a small economic impact in the prediction.

As it is clear from these results, firms' own forecast errors remain significant, despite access to private information. This is unlike our model, in which we have assumed that firms know exactly their available capacity before submitting their bids. Nevertheless, the day-ahead market price and output allocation are computed using firms' committed quantities, even when these differ from their actual ones. Our model tries to capture this fact, namely, that each firm knows exactly how much output it has offered in the day-ahead market. In contrast, a firm can only predict its rival's available capacity, and

[17] We have also used more general specifications, such as a LASSO, with almost identical results.

Variables	(1)	(2)
Public forecast	0.582***	0.070***
	(0.035)	(0.021)
Private forecast		0.657***
		(0.008)
Observations	36,671	36,671
R-squared	0.520	0.826
Mean of the error	0	0
Standard deviation of the error	.18	.11

Table 1: Forecast errors with public versus private information.

Note: The dependent variable is the plant's hourly production normalized by its nameplate capacity. Both regressions include weather data (temperature, wind speed and atmospheric pressure) as well as plant, hour and date fixed effects. The robust standard errors are in parentheses *** $p<0.01$, ** $p<0.05$, * $p<0.1$. One can see that using the plant's own forecast significantly reduces the forecast error, with the R^2 increasing from 0.520 to 0.826. When the private forecast is used, the public forecast is still statistically significant but it has a small impact in the prediction.

hence its quantity offer, with some error.

The day-ahead market, which concentrates the vast majority of trade, is typically followed by a series of balancing markets that operate closer to real time. In general, participation in these markets leads to less favorable prices, and this means that (absent strategic considerations) firms try to avoid imbalances (Hortasu and Puller, 2008). Adding the potential dynamic effects introduced by these sequential markets is out of the scope of this analysis.[18]

3 Equilibrium Characterization

In this section we characterize the Bayesian Nash Equilibria (BNE) of the game in which capacities are private information. It is simple to characterize equilibrium bidding when $\underline{k} > \theta$,, i.e., when either firm can cover total demand regardless of their realized capacities. Hence, Bertrand forces drive equilibrium prices down to marginal costs. Turning attention to the remaining cases, it is useful to start by assuming that $\overline{k} \leq \theta$ (small capacities). In this case, a firm's capacity can never exceed total demand, hence, the low bid is payoff irrelevant. We later analyze the case in which $\overline{k} > \theta$ (large capacities).

[18]See Ito and Reguant (2016) for an empirical analysis.

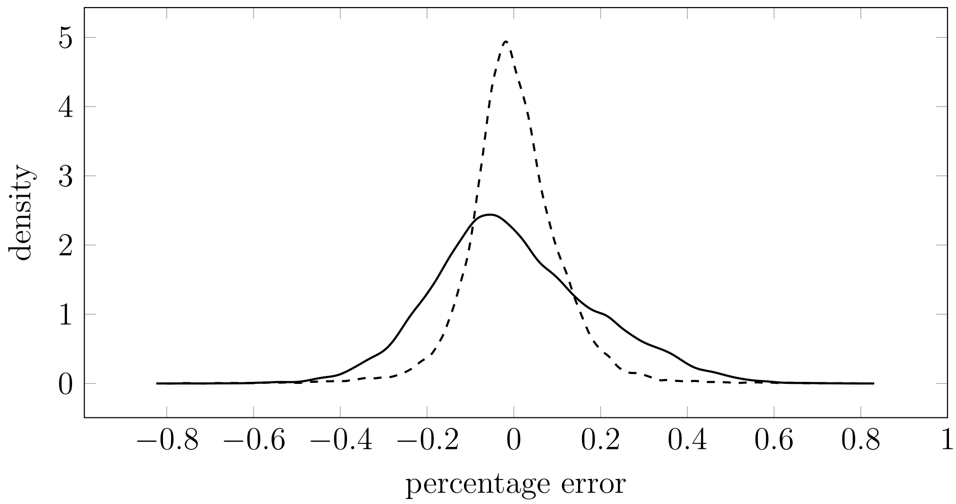

Figure 1: Kernel distribution of the forecasts errors using public (solid) or private information (dashed).

Note: This figure depicts the densities of the forecast errors of the specifications in Table 1. Both distributions are centered around zero, but the standard deviation is larger when only publicly available data are used.

3.1 Small Capacities

We first consider the case of small capacities in the sense that a firm's capacity can never exceed total demand, i.e., $\overline{k} \leq \theta$. Our first Lemma identifies three key properties that any equilibrium must satisfy in this case.

Lemma 1. *If $\overline{k} \leq \theta$,*

(i) Capacity withholding is never optimal, $q_i^*(k_i) = k_i$.

(ii) All Bayesian Nash Equilibria must be in pure strategies.

(iii) The optimal price offer of firm i, $b_i^*(k_i)$, must be non-increasing in k_i.

The first part of the lemma rules out capacity withholding in equilibrium. The is a two-fold reason. First, conditionally on having the low bid, the firm maximizes its output by offering to sell at capacity. Second, conditionally on having the high bid, the firm faces the residual demand. Hence, the firm's output would be unchanged unless withholding constrained the firm from serving the residual demand, $q_i < \theta - k_j$.[19] In that case, its profits would be $(P - c)q_i$, which are below the ones that it could guarantee by bidding all its capacity at P, $(P - c)(\theta - k_j)$.

[19]In particular, offers $q_i \in [\theta - \underline{k}, k_i]$ are payoff equivalent to not withholding at all. In case of indifference, we assume without loss of generality that firms find it optimal not to withhold.

The second part of the lemma rules out non-degenerate mixed strategy equilibria. The underlying reason is simple: a firm's profits at a mixed-strategy equilibrium depend on its realized capacity, which is non-observable by the rival. If the competitor randomizes in a way that makes the firm indifferent between two bids for a given capacity realization, the same randomization cannot make the firm indifferent for other capacity realizations as well. It follows that the equilibria must involve pure strategies.

The last part of the above lemma rules out bids that are increasing in the firm's capacity. When a firm considers whether to reduce its bid marginally, two effects are at play for a given bid of the rival: a profit gain due to the output increase (*quantity effect*), and a profit loss due to the reduction in the market price (*price effect*). On the one hand, the *quantity effect* is increasing in the firm's capacity, as if its bid falls below the rival's, it would sell at capacity rather than just the residual demand. On the other hand, the *price effect* is independent of the firm's capacity as, contingent on bidding higher than the rival, the firm always sells the residual demand. Combining these two effects, the incentives to bid low are (weakly) increasing in the firm's capacity, giving rise to optimal bids that are non-increasing in k_i.[20]

Building on this lemma, we now turn to the characterization of the Bayesian Nash equilibria. We start by describing the pure-strategy asymmetric equilibria, and then move to characterizing the unique pure-strategy symmetric Bayesian Nash equilibrium of the game.

Proposition 1. *If $\overline{k} \leq \theta$, there exist asymmetric Bayesian Nash equilibria in pure-strategies, in all of which $p^* = P$. This market price is sustained by the following price-quantity pairs: $b_i^*(k_i) = P$ and $q_i^*(k_i) = k_i$, while $b_j^*(k_j) \geq \underline{b} > c$ and $q_j^*(k_j) = k_j$, where $\underline{b}$ is low enough so as to make undercutting by firm i unprofitable, $i, j = 1, 2$.*

When firms' capacities never exceed total demand, there exist asymmetric equilibria in which one firm bids sufficiently low so as to discourage its rival from undercutting it. This firm is then forced to maximize its profits over the residual demand by bidding at the highest possible price, P. The low bidder makes higher expected profits than the high bidder, as it sells at capacity rather than the residual demand. Therefore, unless firms can resort to an external correlation device, they are bound to face a coordination

[20]The incentives to bid low are strictly increasing in the firm's capacity if marginally reducing the bid implies a strictly positive probability of increasing the firm's output, i.e., a strictly positive *quantity effect*. This need not be the case if the equilibrium is asymmetric.

problem. This might preclude them from playing the asymmetric equilibria, even if these equilibria allow firms to maximize joint profits.

We now focus on characterizing the symmetric equilibrium. To carry out this analysis it is useful to distinguish two cases. First, assume $\underline{k} \geq \frac{\theta}{2}$ to guarantee that both firms always have enough capacity to jointly serve total demand. Following Lemma 1 part (iii), we already know that the price offers must be non-increasing in capacity. In this case, however, and in contrast with the asymmetric equilibria characterized above, the optimal price offer at a symmetric equilibrium must be *strictly* decreasing in k_i. This result follows from standard Bertrand arguments: equilibrium bidding functions cannot contain flat regions, as firms would otherwise have incentives to slightly undercut those prices. This property allows us to invert $b_j(k_j)$ to write the expected profits of firm i when firm j bids according to a candidate symmetric equilibrium, as follows

$$\pi_i(b_i; k_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j)g(k_j)dk_j.$$

When $k_j < b_j^{-1}(b_i)$, firm i has the low bid and sells up to capacity at the price set by firm j. Otherwise, firm i serves the residual demand and sets the market price at b_i.

In the remaining case, $\underline{k} < \frac{\theta}{2}$, the capacity of both firms might not always be enough to cover total demand. In particular, if $k_i < \theta - \underline{k}$, with probability $G(\theta - k_i)$ the rival's capacity is low enough so that theyfirms cannot jointly cover total demand. Hence, the profit function adds a new term $(P - c)G(\theta - k_i)k_i$.[21] In this case, since both firms produce at capacity regardless of their price offers, the quantity effect does not operate and the optimal price offer is flat as a function of capacity.

Maximizing profits with respect to b_i and applying symmetry, we can characterize the optimal bid at a symmetric equilibrium.

Proposition 2. *If $\overline{k} \leq \theta$, at the unique symmetric Bayesian Nash equilibrium, each firm $i = 1, 2$ offers all its capacity, $q^*(k_i) = k_i$, at a price given by*

$$b^*(k_i) = c + (P - c)\exp\left(-\omega(k_i)\right), \tag{1}$$

for $k_i \geq \hat{k} \equiv \max\left\{\frac{\theta}{2}, \underline{k}\right\}$ where

$$\omega(k_i) = \int_{\hat{k}}^{k_i} \frac{(2k - \theta)g(k)}{\int_k^{\overline{k}}(\theta - k_j)g(k_j)dk_j} dk,$$

and $b^*(k_i) = P$ otherwise.

[21]The profit expression in this case is provided in equation (5) in the Appendix.

First, when $\underline{k} \geq \frac{\theta}{2}$, equation (1) characterizes the optimal price offer for all capacity realizations. In this case, the optimal price offer adds a markup above marginal cost that is strictly decreasing in k_i. In order to provide some intuition, it is useful to implicitly re-write the optimal price offer as follows

$$-\frac{b'^*(k_i)}{b^*(k_i)-c} = \omega'(k_i) = \frac{(2k_i-\theta)g(k_i)}{\int_{k_i}^{\overline{k}}(\theta-k_j)g(k_j)dk_j} \tag{2}$$

This equation captures the incentives to marginally reduce the bid, which, in turn, reflect the trade-off between the *quantity effect* and the *price effect*, as captured by the ratio on the right-hand side of the above equation.

The price effect (on the denominator), or price loss from marginally reducing the bid, is relevant only when the firm is setting the market price, i.e., when the rival firm's capacity is above k_i. In this case, reducing the bid implies that the firm keeps on selling the expected residual demand, $\int_{k_i}^{\overline{k}}(\theta-k_j)g(k_j)dk_j$, but at a lower market price. The quantity effect (on the numerator), or output gain from marginally reducing the firm's bid, is relevant only when the two firms tie in prices, i.e., when both firms have the same capacity k_i, an event that occurs with probability $g(k_i)$. In this case, reducing the bid implies that the firm sells all its capacity rather than just the residual demand, i.e. its output jumps up by $k_i-(\theta-k_i)=2k_i-\theta$.

The quantity effect is weighted by two forces, as captured by the left-hand side of equation (2). On the one hand, the quantity effect is more relevant when the rival's price offer is flatter, since a given reduction in the firm's bid makes it more likely that the firm sells at capacity. On the other hand, the quantity effect is less relevant when the mark-up on the increased sales is smaller.

Consider now the remaining case with $\underline{k} < \frac{\theta}{2}$: in equilibrium, firms bid at P whenever their realized capacities are at or below $\theta/2$. To understand this result, assume that firm i's realized capacity is exactly equal to $\theta/2$. If the rival's capacity were equal or lower than $\theta/2$, reducing the price below P would not allow firm i to increase its profits as both firms would sell all capacity at P regardless of their bids. In contrast, if the rival's capacity exceeded $\theta/2$, firm i would serve the residual demand at its own price. Hence, it would be optimal for firm i to raise the market price up to P, as stated in the Proposition. A similar reasoning implies that bidding at P is optimal when firm i's realized capacity is below $\theta/2$.

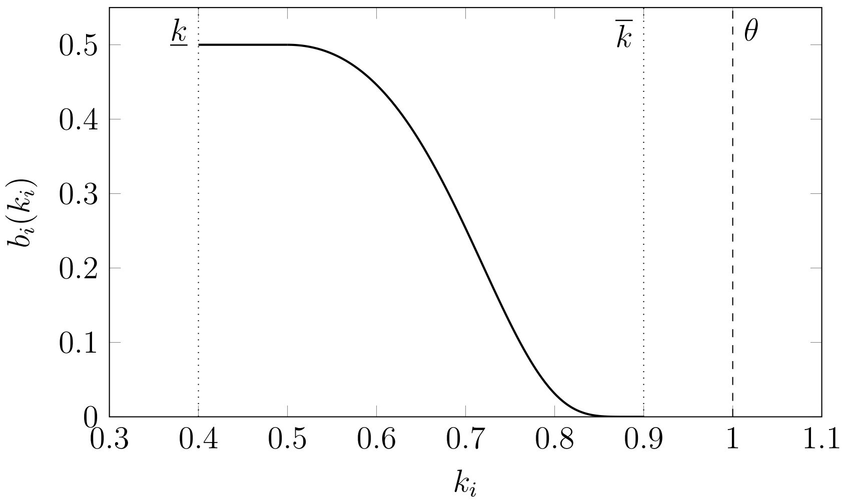

Figure 2: Equilibrium price offer

Note: This figure depicts the equilibrium price offer as a function of k_i when $k_i \sim U[0.4, 0.9]$, with $\theta = 1$, $c = 0$, and $P = 0.5$. One can see that it is flat at P for k_i in $[0.4, 0.5]$, and that it decreases in k_i until it takes the value $c = 0$ at $k_i = \overline{k} = 0.9$.

The optimal bid starts at P for the lowest possible capacity realization and ends at c for the largest one.[22] When $k_i = \underline{k}$, firm j is bigger by construction, so firm i sets the market price with probability one. A price offer below P could never be part of an equilibrium as firm i could sell the same output at a higher price by bidding at P. When $k_i = \overline{k}$, firm j is smaller by construction, so firm i never sets the market price. Hence, the firm's bid has no impact on the price and only the quantity effect matters. Therefore, a price offer above c could never be part of an equilibrium as firm i could expect to sell more output at the same price by bidding at c. Figure 2 depicts the equilibrium price offer as a function of k_i.

Finally, given equilibrium bidding, each firm's expected profits are equal to the minimax when $k_i < \hat{\underline{k}}$, and they are strictly higher otherwise. The reason is simple: a firm can always pretend to be smaller by withholding output and replicating the smaller firm's bid. The fact that firms prefer to offer all their capacity means that larger firms make higher equilibrium profits than the smallest one, whose profits exactly coincide with the minimax. As a result, expected equilibrium market prices are higher than when firms just obtain their minimax.[23]

[22] In this example $\underline{k} < \frac{\theta}{2}$ and, as discussed earlier, a flat region arises that is not present in the case where $\underline{k} \geq \frac{\theta}{2}$.

[23] This result will be useful for Section 4, where we compare equilibrium prices when capacities are private information or unknown to both firms.

3.2 Large Capacities

We now analyze the case of large capacities in the sense that a firm's capacity exceeds total demand with positive probability, $\overline{k} > \theta$. In contrast with the case of small capacities, capacity withholding is now optimal for firms whose capacity exceeds total demand, $k_i > \theta$. Indeed, offering to supply k_i is weakly dominated by offering to supply θ. In any event the firm will never produce more than θ and, conditioning on having the low price, offering k_i instead of θ reduces the chances that the rival's higher price offer sets the market price.[24] For capacity realizations $k_i < \theta$, equilibrium bidding is just as in the case with small capacities.

Proposition 3. *If $\overline{k} > \theta$, in equilibrium, $b_i^*(k_i) = c$ and $q_i^*(k_i) = \theta$ for all $k_i > \theta$, $i = 1, 2$. For $k_i \leq \theta$, Propositions 1 and 2 apply with $G(k_i)$ now adjusted to $G(q_i^*(k_i))$, $i = 1, 2$.*

Essentially, in our model, capacity realizations determine endogenously whether firms compete *à la* Cournot - with firms withholding capacity - or *à la* Bertrand - with firms offering all their capacity at prices above marginal costs.

Combining sections 3.1 and 3.2, we can shed light on two issues: (i) how do prices change when demand goes up, relative to existing capacities?, and (ii) how will prices change as total investment increases?

Regarding the first issue, equilibrium price offers shift out as demand increases since the quantity effect becomes less important, i.e., the quantity loss of being outbid is lower since the residual demand is bigger. Consequently, equilibrium prices go up when demand increases relative to existing capacities. In practice, this implies that market prices will be volatile as the ratio between available capacities and demand moves over time.

Regarding the second issue, an increase in installed capacity κ implies a parallel increase in the capacity bounds $\underline{k} = \beta\kappa + \underline{\varepsilon}$ and $\overline{k} = \beta\kappa + \overline{\varepsilon}$. As this increases the probability that capacity exceeds θ, firms are more likely to bid at c. In turn, this implies that the expected market price smoothly converges towards marginal costs. Eventually, as $\underline{k}$ reaches θ, the market becomes competitive at all times as the equilibrium bid function puts almost all the mass at marginal cost.

This is illustrated in Figure 3. As can be seen in the upper panel, increases in κ shift

[24] If instead of setting the market price at the lowest non-accepted bid, we set it equal to the highest accepted bid, firm i would optimally offer to produce a quantity slightly below total demand, θ, giving rise to the same market price and (almost) the same quantity allocation.

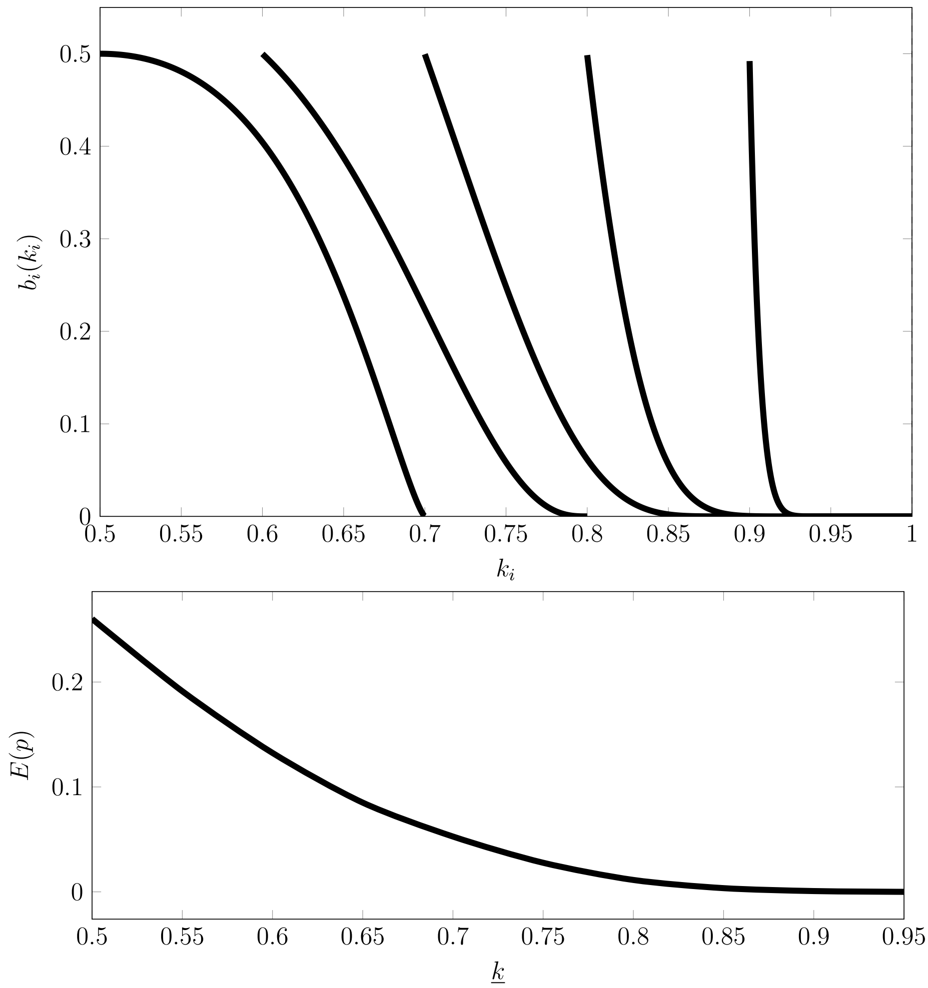

Figure 3: Equilibrium price offers and expected market price as κ increases

Note: The upper panel shows that the equilibrium price offers shift outwards as κ increases. The lower panel shows that the expected market price smoothly goes down as a function of $\underline{k}$, which together with $\overline{k}$, shift our as κ increases. The figures assume $\theta = 1$, $c = 0$, and $P = 0.5$, and $k_i \sim U[\underline{k}, \underline{k} + 0.2]$, for $\underline{k} \in [0.5, 0.95]$.

firms i's equilibrium price offers to the right. This is driven by a stronger *price effect*, as for a given realization of k_i, the rival's capacity is expected to be larger and hence firm i is more likely to set the market price. However, this is more than compensated by the effects of having more installed capacity: since higher capacity realizations become more likely, equilibrium market prices as pushed down, as illustrated in the lower panel of Figure 3.

4 What is the impact of private information?

In this section we want to understand the effect of private information on bidding behaviour and market outcomes. For this purpose, we first characterize equilibrium outcomes under two benchmarks with no private information, either because capacities are publicly known or because they are unknown to both firms prior to bidding. For simplicity, we compare outcomes at the symmetric equilibria.[25]

First, suppose that firms observe realized capacities prior to submitting their bids. Accordingly, firms' bids can be conditioned on realized capacities. The following lemma characterizes the level of profits that can be sustained by symmetric equilibria, either in pure or in mixed strategies.

Lemma 2. *Suppose that realized capacities are publicly known prior to bidding:*

(i) If $k_i < k_j$, there exist symmetric Nash pure-strategy equilibria, resulting in joint profits $(P-c)\theta$. There also exist symmetric Nash mixed-strategy equilibria, resulting in expected joint profits bounded from above by $(P-c)\theta$ and from below by $(P-c)(2\theta - k_i - k_j)$.

(ii) If $k_i = k_j = k$, the unique symmetric Nash equilibrium is in mixed-strategies. It yields expected joint profits $2(P-c)(\theta-k)$.

The game with known capacities allows firms to sustain equilibria in which all their output is sold at P. Just as we described in Section 3, these equilibria are characterized by asymmetric bidding, with one firm bidding at P while the rival bids low enough so as to make undercutting unprofitable. The main difference between this game and the one in which capacities are private information is that firms can use realized capacities (if they are asymmetric) to overcome their coordination problem. For instance, they can now share profits symmetrically by designating the small firm to bid low and the large firm to bid high.[26] Therefore, when realized capacities are publicly known (and asymmetric), there exist symmetric equilibria that allow firms to obtain maximum profits.

The game with publicly known (and asymmetric) capacities also gives rise to a continuum of mixed-strategy equilibria, with firms randomizing their bids between P and c. In one of the extremes, equilibrium profits are lowest when firms do not condition their bidding on realized capacities. In this case, the mixed strategy equilibrium involves no

[25] Comparing the asymmetric equilibria would be uninteresting as the asymmetric equilibria characterized in Proposition 1 can be sustained under all three informational assumptions.

[26] The only exception is when firms' realized capacities are equal. If firms can condition on an external correlation device, they can still share profits symmetrically. Otherwise, the unique symmetric equilibrium involves mixed-strategy pricing, with firms making (weakly) lower expected profits.

firm playing a mass point at P. In the other extreme, equilibrium profits are the highest when one of the two firms plays P with probability almost equal to one, thus converging to the pure-strategy equilibrium characterized above. In turn, this shows that all the mixed strategy equilibria are Pareto-dominated by the pure-strategy equilibrium.

Consider now the case in which firms do not observe realized capacities prior to bidding. For this reason, we need to change the game slightly by assuming that firms' bids are just made of the price at which they are willing to offer all their capacity, once it is realized.[27] The following lemma shows that the unique symmetric equilibrium involves mixed-strategy pricing.

Lemma 3. *If realized capacities (k_i, k_j) are not known prior to bidding, the unique symmetric Bayesian Nash equilibrium involves mixed strategies, with firms randomizing their bids in (c, P). Expected equilibrium joint profits are $2(P - c)(\theta - E[k])$.*

Since bids cannot be conditioned on capacities, in a symmetric equilibrium, both firms would have to either charge equal prices or use the same mixed strategy to randomize their prices. Since the former is ruled out by standard Bertrand arguments, the only symmetric equilibrium involves mixed strategies. Since at P the rival firm is bidding below with probability one, and since all the prices in the equilibrium support yield equal expected profits, it follows that at the unique symmetric equilibrium each firm makes expected profits equal to $(P - c)(\theta - E[k])$.

We are now ready to rank expected prices at the symmetric equilibria across all three information treatments.

Proposition 4. *The comparison of the symmetric Bayesian Nash equilibria in the games in which capacities are unknown, private information, or publicly known, shows that:*

(i) The lowest expected prices are obtained with unknown capacities.

(ii) The highest expected prices are obtained with publicly known capacities.

The proposition above shows that the more information firms have, the higher the expected prices they can obtain at a symmetric equilibrium. When capacities are private information, the fact that bidding incentives differ across firms allows them to avoid fierce competition, but not as much as if both capacities were known: large (small) firms find

[27] In any event, firms would not find it optimal to withhold output below their expected capacity.

it in their own interest to bid low (high), but not as low (high) as if they knew with certainty that the rival firm were bidding above (below). When capacities are unknown to both firms, firms face fully symmetric incentives and hence they end up competing fiercely. This explains why private information leads to higher prices than in the case with unknown capacities, but lower than when capacities are publicly known. This suggests that firms would be better off if they could exchange their private information regarding their available capacities. For the same reason, in the context of electricity markets, system operators should avoid publishing individual firm information even if, in practice, they collected it to construct an aggregate forecast.

The above results serve to shed some light on the relationship between the precision of information and equilibrium bidding. In our model, when firms get a very precise signal about the rival's capacity, equilibrium profits converge to those with symmetric and known capacities, for which the symmetric equilibrium involves mixed strategies that give rise to very low profits (i.e., reaching the minimax).

However, this result might be misleading as it only applies to instances when firms are ex-post symmetric, an event that occurs with zero probability. More generally, the model developed so far does not allow to disentangle the effects of improved information precision from those of increased symmetry: as the precision of the signal increases, not only firms become better informed about the rival's capacity but also their capacities are more likely to become ex-post symmetric. Since increased symmetry leads to more competitive outcomes, this latter effect confounds the true impact of information on bidding behavior.

This question has already been addressed in other contexts. Spulber (1995) showed that introducing asymmetric information on firms' costs in the standard price competition model leads to higher prices. However, the opposite conclusion is reached when comparing a game with private information on costs versus one in which costs are known but they are stochastic, using the same distribution (Hansen, 1988).[28] That is, private information on costs mitigates market power, in line with our model's predictions regarding the impact of private information on capacities.

Accordingly, in order to understand the effects of information precision in our model,

[28] Lagerlöf (2016) shows that the convergence between both models occurs not because of an increase in the precision of the signal that firms receive but, rather, due to an increasing affiliation of the costs shocks.

one would need to work with ex-ante asymmetric capacities, as we do in Section 5.2. That analysis allows us to conclude the following. Suppose that k_i is uniformly distributed in $[\underline{k}_i, \overline{k}_i]$, with $\overline{k}_1 < \underline{k}_2$, i.e., firm 1 is always smaller than firm 2. In this case, there cannot exist equilibria similar to those in Proposition 2 as there is no uncertainty on which firm has the small capacity and hence the high bid. As a consequence, the only pure strategy Bayesian Nash equilibria are those in Proposition 1, with $p^* = P$. In words, the introduction of a small amount of uncertainty around asymmetric capacities has no impact on bidding behavior or market outcomes. However, adding more uncertainty so that firms' capacity intervals overlap, $\overline{k}_1 > \underline{k}_2$, gives rise to equilibria in which firms' bids are a function of realized capacities (along the lines of Proposition 2). As in Proposition 4, the expected market price starts falling below P as the forecasts about the rival's capacity become more noisy.

5 Extensions and Variations

In this section we consider extensions and variations of the baseline model. First, we consider changes in the market structure: we allow for more than two symmetric firms, and we introduce asymmetries across firms. Second, we consider changes in the market design: we characterize equilibrium bidding under the discriminatory auction, and assess the effects of banning capacity withholding.

5.1 N Firms

In this section we extend our equilibrium analysis to accommodate an arbitrary number of symmetric firms, $N \geq 2$. For simplicity, we focus our discussion on the case in which the capacity of all firms is always necessary to cover demand. As a result, all firms but the one with the highest bid (and the smallest capacity) will sell at capacity. This means that from the point of view of firm i the N-firm problem can be reinterpreted as if each firm was only facing the smallest competitor.

We introduce some additional notation. Let k_{-i} be the minimum capacity among those of firm i's rivals, i.e., $k_{-i} = \min_{j \neq i} k_j$. As usual, its cumulative distribution function and density are

$$\begin{aligned}
\Phi(k_{-i}) &= 1 - (1 - G(k_{-i}))^{N-1}, \\
\varphi(k_{-i}) &= (N-1)\, g(k_{-i})\, (1 - G(k_{-i}))^{N-2}.
\end{aligned}$$

The following result extends Proposition 2.

Proposition 5. *When $(N-1)\overline{k} \leq \theta$, at the unique symmetric Bayesian Nash equilibrium, each firm $i = 1, .., N$ offers all its capacity, $q^*(k_i) = k_i$, at a price given by*

$$b^*(k_i) = c + (P - c) \exp\left(-\omega\left(k_i\right)\right),$$

for $k_i \geq \hat{\underline{k}} \equiv \max\left\{\frac{\theta}{N}, \underline{k}\right\}$ where

$$\omega\left(k_i\right) = \int_{\underline{k}}^{k_i} \frac{\left(2k + \int_k^{\overline{k}} (N-2)\, kg(k)dk - \theta\right)\varphi(k)}{\int_k^{\overline{k}}\left(\theta - k_j - \int_{k_j}^{\overline{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j} dk,$$

and $b^*(\underline{k}) = P$ otherwise.

As compared to the solution in the duopoly case, N enhances the quantity effect because the loss in production from marginally increasing the bid is higher the more competitors there are in the market. At the same time the price effect is reduced because the firm only benefits from increasing the bid through the residual demand, which is now smaller. Both effects imply that the optimal price offer goes down with N and so does the equilibrium price.

5.2 Asymmetric Firms

We now discuss the case in which firms have different nameplate capacities. In particular, firm i has capacity $k_i = \beta\kappa_i + \varepsilon_i$ with $\kappa_1 > \kappa_2$. For simplicity, we assume that errors are uniformly distributed in a common interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, so that the capacity of firm i is uniformly distributed between $\underline{k}_i = \beta\kappa_i + \underline{\varepsilon}_i$ and $\overline{k}_i = \beta\kappa_i + \overline{\varepsilon}_i$ for $i = 1, 2$. As in Section 3, we start with the case in which the two firms' capacities are small, $\overline{k}_1 \leq \theta$.

Proposition 6. *Assume that k_i is uniformly distributed in $[\underline{k}_i, \overline{k}_i]$. If $\overline{k}_1 \leq \theta$, when firms have ex-ante asymmetric capacities, in equilibrium, each firm offers all its capacity, $q_i^*(k_i) = k_i$ for $i = 1, 2$, and*

(i) The pure strategy Bayesian Nash equilibria characterized in Proposition 1 continue to exist.

(ii) Furthermore, if $\overline{k}_2 \geq \underline{k}_1$, there also exists an equilibrium in which price offers are characterized as

$$b_1^*(k_1) = \begin{cases} b^*(k_1) & \textit{if } k_1 < \overline{k}_2, \\ c & \textit{otherwise,} \end{cases} \qquad b_2^*(k_2) = \begin{cases} P & \textit{if } k_2 < \underline{k}_1, \\ b^*(k_2) & \textit{otherwise,} \end{cases}$$

where

$$b^*(k_i) = c + (P - c)\exp\left(-\omega(k_i)\right), \tag{3}$$

for $k_i \geq \hat{\underline{k}}_1 \equiv \max\left\{\frac{\theta}{2}, \underline{k}_1\right\}$ *and*

$$\omega(k_i) = \int_{\hat{\underline{k}}_1}^{k_i} \frac{(2k-\theta)}{\int_k^{\overline{k}_2}(\theta - k_j)dk_j} dk,$$

and $b^*(k_i) = P$ *otherwise.*

When the capacity intervals do not overlap, the equilibria are identical to the ones characterized in Proposition 1, with one firm setting the market price at P and the other one choosing a sufficiently low bid. There cannot exist a symmetric equilibrium as the one in Proposition 2, as it relies on firms being uncertain about the identity of the large firm and, therefore, about the identity of the low bidder.

In contrast, when the capacity intervals overlap, this uncertainty reemerges for capacities in the range $[\hat{\underline{k}}_1, \overline{k}_2]$. Over this interval, the equilibrium price offers resemble those in Proposition 2, with firms pricing at P for $k_i = \hat{\underline{k}}_1$ and at c for $k_i = \overline{k}_2$. For smaller capacity realizations, firm 2 bids at P. Instead, for higher capacity realizations, firm 1 bids at c. As a result, both price offers are continuous in the realized capacities.

The next corollary shows that the optimality of withholding implies that these equilibria survive in the large capacities case.

Corollary 1. *If* $\overline{k}_1 > \theta$, *in equilibrium each firm offers* $q_i^*(k_i) = \min\{\theta, k_i\}$ *and prices according to Proposition 1, where the relevant thresholds* $\underline{k}_1$ *and* $\overline{k}_2$ *become* $q_1^*(\underline{k}_1)$ *and* $q_2^*(\overline{k}_2)$, *respectively.*

Parallel to the ex-ante symmetric capacities case, firms always find it optimal to withhold capacity whenever their realized capacity exceeds θ. As a result, firms behave in equilibrium as if their capacities were capped, with a mass at θ.

It is important to notice that the characterization of this equilibrium hinges on the density of each firm being identical in the range of capacity overlap, thanks to the assumption of uniformly and identically distributed idiosyncratic shocks. This guarantees that the two first order conditions that characterize optimal bidding are identical, allowing us to conclude that the equilibrium price offers are symmetric. While we do not provide a characterization for generic distribution functions, we conjecture that the nature of the equilibrium would remain the same.

5.3 Capacity Withholding is not Allowed

In Section 3.2 we showed that the symmetric equilibrium involves capacity withholding whenever $k_i > \theta$. From a regulatory perspective, would consumers be better off with a ban on capacity withholding? To address this question we now characterize equilibrium bidding when capacity withholding is not possible and compare it with the one reported in Proposition 2.

The constraint $q_i = k_i$ is clearly not binding in case of small capacities, $\overline{k} \leq \theta$, as firms never find it optimal to withhold capacity even if allowed (Lemma 1). In contrast, banning capacity withholding in cases with $\overline{k} > \theta$ has a dramatic impact on bidding incentives. First, conditionally on having the high bid, a firm produces nothing if its rival's capacity is at or above θ. Second, the bid of a firm whose capacity exceeds θ is payoff relevant even if it is the low one, as in this case the firm serves total demand at its own bid. The former effect intensifies competition, whereas the second induces firms to charge higher prices.

To describe the equilibrium when capacity withholding is not allowed, we split the characterization in two cases, depending on whether the firm's realized capacity is above or below θ.

Lemma 4. *Assume $\overline{k} > \theta$. There does not exist a Bayesian Nash Equilibrium in pure strategies when capacity withholding is not allowed. Furthermore, in any equilibrium, for $k_i > \theta$, firm i randomizes its bid in a support $\left[\underline{b}, \bar{b}\right]$ independently of k_i, where $\underline{b} > c$ and $\bar{b} < P$.*

If $k_i > \theta$, firm i is never capacity constrained. Since its expected profits do not depend on its realized capacity, its optimal bid at a candidate pure-strategy equilibrium is the same for all capacity realizations above θ. However, this would give rise to ties with positive probability but this is thus ruled out by standard Bertrand-Edgeworth arguments. More specifically, ties cannot be part of an equilibrium as firms would be better off by slightly undercutting any price above marginal cost in order to sell more output with only (if any) a slight reduction in the price. Furthermore, tying at marginal cost is ruled out as firms could make positive profits by selling the expected residual demand at P. Thus, at a symmetric equilibrium, firms must randomize their bids for all capacity realizations above θ.

The previous argument implies, of course, that the symmetric Bayesian Nash Equilibrium of the game must be in mixed strategies, as at least for $k_i > \theta$ firms randomize their bids. One distinctive feature of this equilibrium is that the upper bound of the price support does not go all the way up to P. The reason is that when firms have capacity $k_i > \theta$, they face a downward sloping residual demand, induced by the downward sloping bid function of the rival when its capacity realization is below θ. We now turn to characterizing the equilibrium in this case. The next proposition describes the optimal bid, and Figure 4 illustrates it.

Proposition 7. *Assume* $\overline{k} > \theta$. *In the unique symmetric Bayesian Nash Equilibrium without capacity withholding, if* $k_i < \theta$, *the optimal bid for firm* i *is*

(i) $b^*(k_i; \underline{k}, \theta)$ *for* $k_i \in \left[\underline{k}, \hat{k}\right]$ *and* $k_i \in \left[\tilde{k}, \theta\right]$, *as defined in* (1).

(ii) $\hat{b}(k_i; \underline{k}, \theta)$ *for* $k_i \in \left(\hat{k}, \tilde{k}\right)$, *strictly decreasing in* k_i *and strictly lower than* $b^*(k_i; \underline{k}, \theta)$.

(iii) $b_i \sim F(b_i)$ *with density* $f(b_i)$ *in a support* $[\underline{b}, \bar{b}]$.

The thresholds $\hat{k}$ *and* $\tilde{k}$ *are implicitly defined as* $b^*(\hat{k}; \underline{k}, \theta) = \bar{b}$ *and* $b^*(\tilde{k}; \underline{k}, \theta) = \underline{b}$, *where* $\underline{b}$ *and* $\bar{b}$ *are defined in Lemma 4.*

The optimal bid when k_i belongs to either $\left[\underline{k}, \hat{k}\right]$ or $\left[\tilde{k}, \theta\right]$ is similar to the one when capacity withholding is allowed. The sole difference is that, from firm i's point of view, firm j's relevant capacities now range from $\underline{k}$ to θ given that firm i's profits are constant when $k_j > \theta$. In particular, for such capacities, firm j randomizes its bid in the support $(\underline{b}, \bar{b})$ and, thus, price offers are bounded from above by $b^*(\hat{k}; \underline{k}, \theta) = \bar{b}$ and from below by $b^*(\tilde{k}, \underline{k}, \theta) = \underline{b}$. Hence, if $k_j > \theta$, firm i does not produce anything if k_i belongs to $\left[\underline{k}, \hat{k}\right]$, while firm i sells at capacity at the price set by firm j if k_i belongs to $\left[\tilde{k}, \theta\right]$. It follows that firm i's marginal profits are zero whenever $k_j > \theta$ and, hence, its bidding incentives are equal to those in Proposition 2 with $\overline{k} = \theta$.

This result is in contrast to the case where $k_i \in \left[\hat{k}, \tilde{k}\right)$. For these realizations, firm i might have the low or the high bid depending on the bid chosen by firm j when playing its mixed strategy. In particular, firm i's incentives to bid low are now stronger as compared

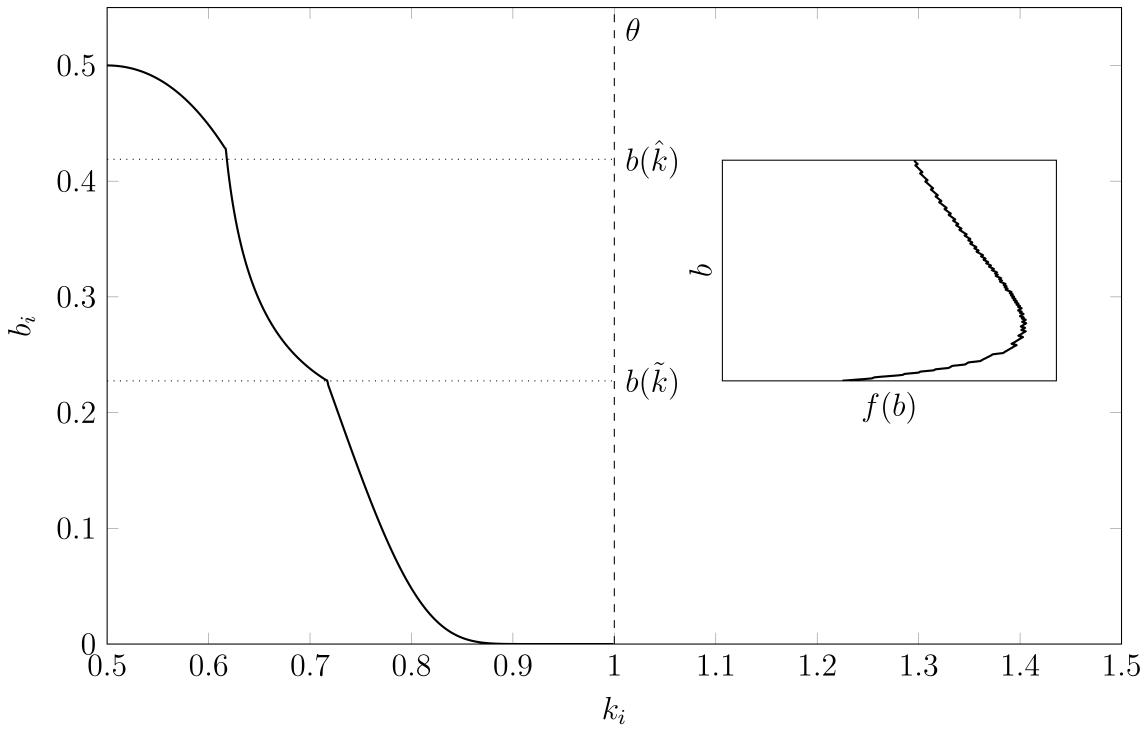

Figure 4: Equilibrium price offers when capacity withholding is not allowed.

Note: The figure depicts the equilibrium price offer. The equilibrium involves a pure-strategy for capacities below θ and a mixed strategy for capacities above θ (the price density is depicted on the right). Parameter values $k_i \sim U[0.5, 1.1]$, with $\theta = 1$, $c = 0$, and $P = 0.5$.

to those in the withholding case, given that by reducing its price offer it can outbid the rival for a larger range of capacity realizations, including $k_j > \theta$.

The previous equilibrium bidding function is not monotonic in k_i, particularly around θ. The optimal bid converges to c to the left of θ as the firm is certain to be selling at capacity at the price set by the rival. In contrast, the bid jumps above c when $k_i > \theta$, as the firm is aware that its bid is always payoff relevant.

Allowing $\overline{k}$ to increase above θ shows how the equilibrium bid schedules approach the competitive outcome. Suppose that capacities are uniformly distributed in $\left[\underline{k}, \overline{k}\right]$, and consider moving the whole capacity support to the right. For capacity realizations above θ, the equilibrium mixed strategy puts increasingly more weight on the lower bound of the price support, which converges towards c. In turn, the range $\left(\hat{k}, \tilde{k}\right)$ widens up. This process continues until $\underline{k}$ reaches θ, in which case the equilibrium bid functions become flat at marginal costs. Figure 5 depicts this process of convergence towards the competitive outcome.

We can now assess the effect on consumers of banning capacity withholding comparing

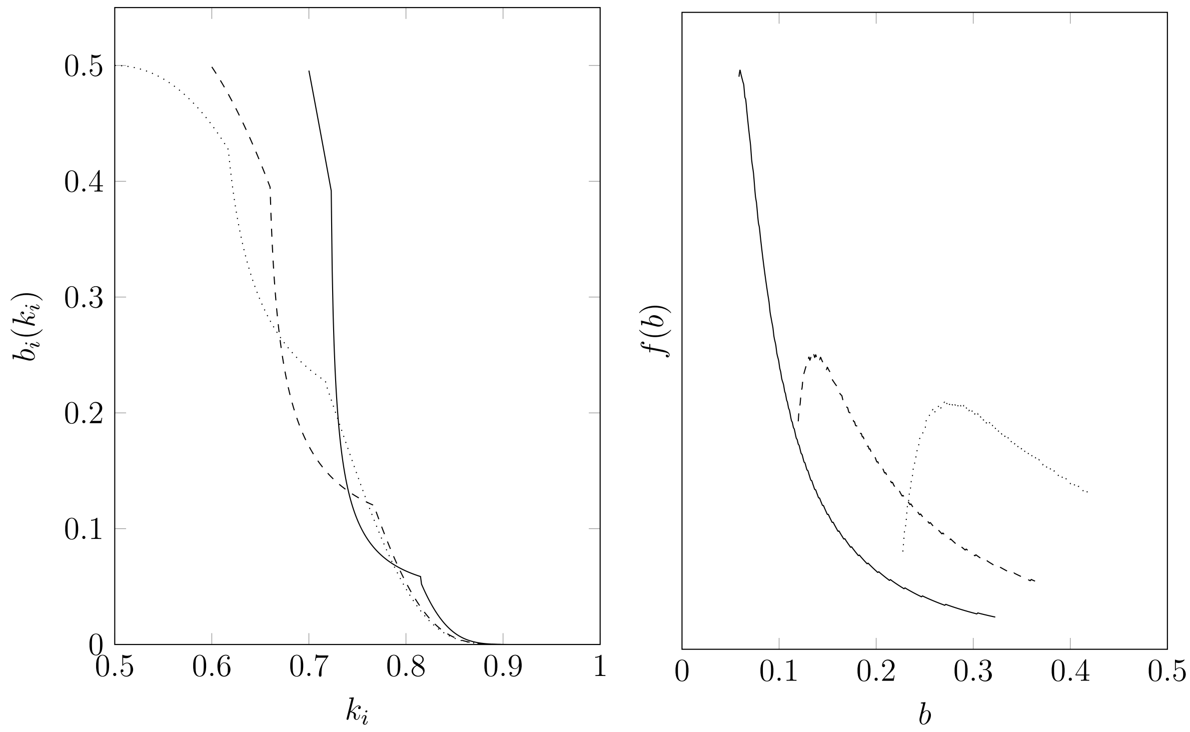

Figure 5: Equilibrium bids and probability density when $k_i \sim U[0.5, 1.1]$ (dotted), $k_i \sim U[0.6, 1.2]$ (dashed) and $k_i \sim U[0.7, 1.3]$ (solid), with $\theta = 1$, $c = 0$, and $P = 0.5$.

Note: The figure shows the equilibrium price offers (left panel) and price distributions (right panel). As κ increases, the price offers move downwards, while the densities put more weight on lower prices.

the equilibria characterized in Propositions 2 and 7. There are two forces operating in opposite directions. On the one hand, for $k_i > \theta$, capacity withholding yields lower bids as firms offer to produce θ at marginal cost. On the other hand, for $k_i \leq \theta$, firms' bids are weakly lower without capacity withholding, and strictly so for capacity realizations in the range $\left(\hat{k}, \tilde{k}\right)$. Numerical results like those illustrated in Figure 6 suggest that the first effect dominates. In particular, when capacities are uniformly distributed, banning capacity withholding gives rise to higher expected prices.

6 Concluding Remarks

In this paper we have analyzed equilibrium bidding in multi-unit auctions when bidders' production capacities are private information. We have allowed changes in capacity to shape the bidding functions, both through changes in the prices and the quantities offered by firms. This is unlike other papers in the literature which typically assume that the private information is on costs (or bidders' valuations) and which, with few exceptions, do not allow bidders to act on both the price and quantity dimensions.

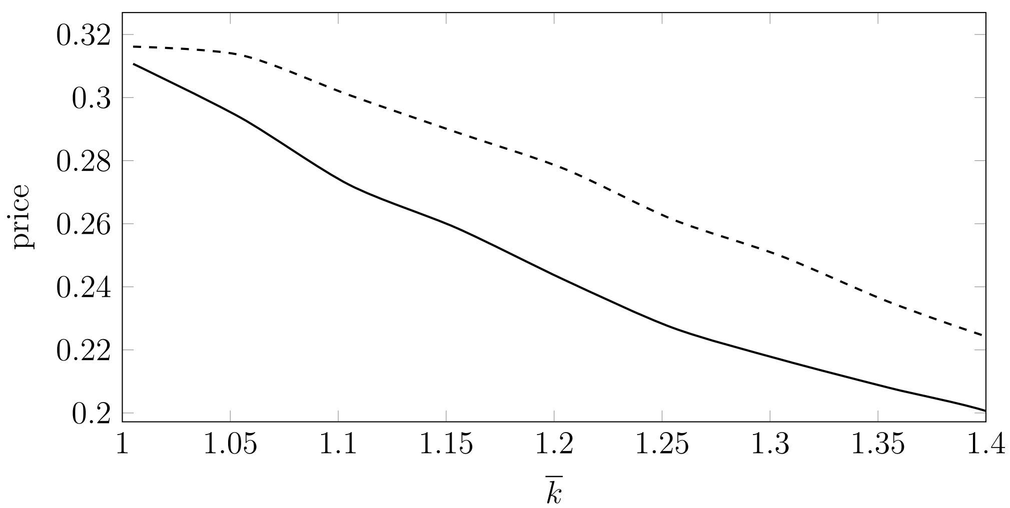

Figure 6: Equilibrium prices for $\underline{k} = 0.5$ and $\overline{k} > 1$ when withholding is possible (solid line) or not allowed (dashed line)

Note: This figure depicts the expected market price as a function of $\overline{k}$ under the assumptions $\theta = 1$, $c = 0$, and $P = 0.5$, and $k_i \sim U[\underline{k}, \underline{k}]$, for $\underline{k} = 0.5$ and $\overline{k} \in [1.05, 1.4]$. It shows that expected market prices are lower when capacity withholding is allowed.

We have shown that the nature of private information and the strategies available to firms have a key impact on equilibrium behavior. As compared to cost shocks, equilibrium prices are more elastic to capacity shocks. The reason is two-fold: firms' price offers tend to be steeper in their private information, and firms find it optimal to offer more output at lower prices when they receive a positive capacity shock. We have also shown that firms tend to exercise less market power the greater the capacity uncertainty. As a consequence, under private information on capacities, firms can obtain higher profits than when capacities are unknown, but less than when capacities are common knowledge.

Even though our model applies to a range of auction settings in which bidders possess private information about their capacities, we have motivated it in the context of electricity markets. Understanding competition among renewables is of first order importance to guide policy making in this area.

Our paper provides some key lessons about the future performance of electricity markets. First, our equilibrium characterization demonstrates that renewables will not in general make electricity markets immune to market power. Rather, firms will keep on exercising market power by raising their bids when their capacities are not large enough, and by withholding their output otherwise. The fact that the price offers are decreasing in firms' capacities implies that mark-ups will be lower at times with more available capacity, leading to price dispersion both within as well as across days, depending on weather conditions.

Renewables introduce a trade-off between price levels and price volatility. As we have shown, renewables tend to mitigate market power as compared to conventional technologies, as the former have unknown capacities while the capacities of the latter are known. However, to the extent that the marginal costs of fossil-fuels are less uncertain than the availability of renewables, market prices will also tend to be more volatile. The prevalence of positive mark-ups implies that the price-depressing effects of renewables will not be as pronounced as predicted under the assumption of perfect competition.

Renewables will have a stronger price depressing effect in the long-run as installed renewable capacity goes up. The reduction in expected prices as a function of total investment will not be linear, but it will rather be smoother at the late stages of the energy transition. Our model predicts that differences across renewable technologies (e.g. solar versus wind) will give rise to different market power impacts. For instance, solar will give rise to less market power than wind, to the extent that solar forecasts are typically less precise. Introducing correlation between demand and the expected availability of each technology would also highlight differences across technologies as, in contrast to solar power, wind tends to depress prices when demand is low but has little effect on prices when demand is high. Competition among renewables could also be affected by portfolio effects, as firms typically own a variety of technologies whose joint distribution will affect firms' optimal bidding strategies. In sum, future electricity markets will depict large price differences across the day and across the year, reflecting differences in weather conditions and the associated differences in firms' ability to charge positive mark-ups. As compared to conventional technologies, renewables help mitigate market power.

The aim of this paper has not been to identify the optimal design of future electricity markets, but rather to analyze their future performance if the current mechanisms remain unchanged. It has shown that deploying renewables without further market design changes will not be enough to achieve efficient outcomes. Regulators will have to rely on other instruments or other market designs if they aim at fully eliminating market power.

Notably, our analysis has taken capacities as given. A related issue that deserves further research is whether the current mechanisms will induce the desired investments. This paper has provided a first step in this highly policy relevant research agenda.

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A Proofs

Proof of Lemma 1: For part (i) of the lemma, suppose that firm j chooses a bid according to a distribution $F_j(b_j, q_j|k_j)$. Profits for firm i can be written as

$$\pi_i(b_i, q_i, F_j|k_i) = \int_{k_j} \int_{(b,q \le \theta - q_i)} (P-c)q_i dF_j(b,q|k_j) + \int_{(b,q>\theta-q_i)} [(b-c)q_i \Pr(b_i \le b) \\ + (b_i - c)(\theta - q)\Pr(b_i > b)]\, dF_j(b,q|k_j)g(k_j)dk_j.$$

The above equation is increasing in q_i, indicating that the firm maximizes profits by choosing $q_i^*(k_i) = k_i$. In what follows we simplify the notation by eliminating q_i from the profit function π_i and by indicating that the randomization is only over prices, $F_i(b_i|k_i)$.

For part (ii), consider, towards a contradiction, two bids b_i and $b_i' > b_i$ for which firm i randomizes. Then, it must be that firm i is indifferent between both and, thus,

$$\pi_i(b_i', F_j|k_i) - \pi_i(b_i, F_j|k_i) = \int_{\max\{\underline{k}, \theta - k_i\}}^{\overline{k}} \int_b \{(b-c)k_i [\Pr(b_i' \le b) - \Pr(b_i \le b)] \\ +(\theta - k_j)[(b_i' - c)\Pr(b_i' > b) - (b_i - c)\Pr(b_i > b)]\}\, dF_j(b|k_j)g(k_j)dk_j = 0. \tag{4}$$

Consider first the case in which $\underline{k} \ge \theta - k_i$ so that the limit of integration does not depend on k_i. Since j cannot condition the strategy on k_i, then $F_j(b_j|k_j)$ must be such that the previous expression holds for all k_i. Hence, either $b_j(k_j) = c$ for all k_j, in which case $F_j(b_j|k_j)$ would be a degenerate mixed strategy, or

$$\int_{k_j} \int_b [\Pr(b_i' \le b) - \Pr(b_i \le b)]\, dF_j(b|k_j)g(k_j)dk_j = 0.$$

By Bertrand arguments, F_j cannot contain gaps in the support and, therefore, this cannot occur. Given that the second part of equation (4) does not depend on k_i this leads to a contradiction.

Consider now the case in which $\underline{k} < \theta - k_i$, where different values of k_i affect the limit of integration in equation (4). However, for $k_j \in [\underline{k}, \theta - k_i]$, since there is not enough capacity to cover the demand and the market price is set at P, profits are equal to $(P-c)k_i$ regardless of the bid and the difference in profits is, in that case, equal to 0. As a result only the effect that arises when $\underline{k} \ge \theta - k_i$ matters and the same proof follows.

Regarding part (iii) of the lemma, using the previous result we can focus on firm j choosing a pure strategy. As a result, it is enough to show that the function $\pi_i(b_i, b_j(k_j)|k_i)$

has non-increasing differences in b_i and k_j. Using the previous expression and taking the derivative with respect to k_i we have

$$\frac{\partial\left[\pi_i(b_i', b_j(k_j)|k_i) - \pi_i(b_i, b_j|k_i)\right]}{\partial k_i} =$$
$$\int_{\max\{\underline{k}, \theta - k_i\}}^{\overline{k}} (b_j(k_j) - c)k_i\left[\Pr(b_i' < b_j(k_j)) - \Pr(b_i < b_j(k_j))\right] g(k_j)dk_j \leq 0.$$

In words, larger firms gain (weakly) less from increasing their bids. Hence, the optimal bid function is non-increasing in k_i. □

Proof of Proposition 1: See Fabra et al. (2006).

Proof of Proposition 2: Consider equilibria with the following shape:

$$\tilde{b}_j(k_j) = \begin{cases} P & \text{if} \quad k_j < \hat{\underline{k}}, \\ b_j(k_j) & \text{if} \quad k_j \geq \hat{\underline{k}}. \end{cases}$$

Notice that $b_j(k_j)$ is strictly decreasing k_j in the symmetric equilibrium. Towards a contradiction, suppose that this is not the case. From Lemma 1, this implies that there is a region $[k_a, k_b]$ such that both firms choose the same bid, $b_i = b_j$. At least one of the firms would not sell all its capacity for some capacity realizations. The standard Bertrand argument implies that this cannot be part of an equilibrium, as the firm that sells below capacity could increase its profits by slightly undercutting the competitor in order to sell at capacity.

We start by showing that $\hat{\underline{k}} = \max\left\{\frac{\theta}{2}, \underline{k}\right\}$. First, it cannot be lower than $\theta/2$. Argue by contradiction and suppose $\hat{\underline{k}} < \theta/2$ or, rearranging, $\hat{\underline{k}} < \theta - \hat{\underline{k}}$. This cannot part of a symmetric equilibrium since for $k_i \in \left(\hat{\underline{k}}, \theta - \hat{\underline{k}}\right)$ aggregate capacity is not enough to cover total demand, implying that the best response of firm i includes P, a contradiction.

We now show that $\hat{\underline{k}}$ cannot be greater than $\theta/2$ if $\theta/2 > \underline{k}$. Argue by contradiction and suppose $\tilde{k} > \theta/2$. This cannot be part of a symmetric equilibrium since for $k_i \in \left(\theta/2, \tilde{k}\right)$ firm i would be better off undercutting P. If the rival firm's capacity falls in the interval $k_j \in \left(\theta - k_i, \tilde{k}\right)$, the two firms would tie at P and each one would sell below capacity. By slightly undercutting P, expected profits would increase by $(P - c)\left(G\left(\tilde{k}\right) - G(\theta - k_i)\right)\left(k_i - \frac{\theta}{2}\right)$. It follows that we must have $\tilde{k} = \theta/2$.

In turn, note that this implies that we only have ties at P when both firms have capacity below $\theta/2$ so that aggregate capacity is not enough to cover total demand. Otherwise, when at least one firm is selling below capacity, ties at P can never occur.

Expected profits are

$$\pi_i\left(b_i, b_j | k_i\right) = \left(P - c\right) k_i G\left(\theta - k_i\right) + \int_{\max(\theta - k_i, \underline{k})}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j$$
$$+ \int_{b_j^{-1}(b_i)}^{\overline{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j, \tag{5}$$

and the first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{\partial \pi_i}{\partial b_i} = b_j^{-1\prime}(b_i) g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\overline{k}} (\theta - k_j) g(k_j) dk_j = 0. \tag{6}$$

Furthermore, around the candidate equilibrium, the profit function is strictly concave.

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$\frac{1}{b_i'(k_i)} g(k_i)(b_i(k_i) - c)(2k_i - \theta) + \int_{k_i}^{\overline{k}} (\theta - k_j) g(k_j) dk_j = 0. \tag{7}$$

If $k_i \leq \theta/2$, the first term in the first order condition (7) is always positive, hence it is optimal to bid at P. If $k_i > \theta/2$, the first term of the first order condition (7) is negative and the second term is positive, so an interior solution exists. Indeed, the first order condition takes the form

$$b_i'(k_i) + a(k_i) b_i(k_i) = c a(k_i),$$

where

$$a(k) = \frac{(2k - \theta) g(k)}{\int_k^{\overline{k}} (\theta - k_j) g(k_j) dk_j}. \tag{8}$$

If we multiply both sides by $e^{\int_{\underline{\hat{k}}}^k a(s)ds}$ and integrate from $\underline{\hat{k}}$ to k_i we obtain

$$\int_{\underline{\hat{k}}}^{k_i} \left(e^{\int_{\underline{\hat{k}}}^k a(s)ds} b_i'(k) + a(k) e^{\int_{\underline{\hat{k}}}^k a(s)ds} b_i(k) \right) dk_i = c \int_{\underline{\hat{k}}}^{k_i} a(k_i) e^{\int_{\underline{\hat{k}}}^k a(s)ds} dk_i.$$

We can now evaluate the integral as

$$e^{\int_{\underline{\hat{k}}}^k a(k)dk} b_i(k) \Big]_{\underline{\hat{k}}}^{k_i} = c e^{\int_{\underline{\hat{k}}}^k a(s)ds} \Big]_{\underline{\hat{k}}}^{k_i}.$$

This results in

$$e^{\int_{\underline{\hat{k}}}^{k_i} a(k)dk} b_i(k_i) - b_i(\underline{\hat{k}}) = c e^{\int_{\underline{\hat{k}}}^{k_i} a(k)dk} - c.$$

Solving for $b_i(k_i)$ we obtain

$$b_i(k_i) = c + A e^{-\int_{\underline{\hat{k}}}^{k_i} a(k)dk} = c + A e^{-\omega(k_i)},$$

where $A \equiv b_i(\hat{\underline{k}}) - c$ and $\omega(k_i) \equiv \int_{\hat{\underline{k}}}^{k_i} a(k)dk$.

A necessary condition for an equilibrium is that equilibrium profits are at or above the minimax, which the firm can obtain by bidding at P. Hence, a necessary and sufficient condition for equilibrium existence is that

$$\pi_i(b_i, b_j|k_i) \geq (P-c)\,k_i G(\theta - k_i) + \int_{\max(\theta-k_i,\underline{k})}^{\overline{k}} (P-c)(\theta-k_j)g(k_j)dk_j. \tag{9}$$

Hence, to rule out deviations to P, we now need to prove that minimax profits increase less in k_i as compared to equilibrium profits. First, suppose that $k_i < \hat{\underline{k}}$. Since each firm optimally chooses bids $b(k_i) = b(\hat{\underline{k}}) = P$ the minimax is reached with equality. As a result, $A = P - c$.

Second, suppose that $k_i \in \left[\hat{\underline{k}}, \theta - \underline{k}\right)$. The derivative of the minimax with respect to k_i is

$$(P-c)G(\theta - k_i),$$

whereas, from (5), the derivative of the profit function, using the Envelope Theorem, can be computed as

$$(P-c)G(\theta-k_i) + \int_{\theta-k_i}^{b_j^{-1}(b_i)} (b_j(k_j)-c)k_i g(k_j)dk_j.$$

Clearly, this derivative is greater than that of the minimax.

Finally, suppose $k_i \geq \theta - \underline{k}$. The derivative of the minimax is

$$(P-c)\left(G(\theta-k_i) - g(\theta-k_i)k_i\right).$$

The derivative of profits is

$$(P-c)\left(G(\theta-k_i) - g(\theta-k_i)k_i\right) + \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j)-c)k_i g(k_j)dk_j.$$

Again, this derivative is greater than that of the minimax.

It follows that deviations to P are not profitable since equilibrium profits are always strictly greater than the minimax, except for $k_i \leq \theta/2$ when equilibrium profits are exactly equal to the minimax.

Finally, we need to verify that the candidate equilibrium, indeed, maximizes profits for each of the firms. From the first order condition in (6) we can compute the second derivative of the profit function of firm i, when firm j uses a bidding function $b_j(k_j)$ as

$$\frac{g(b_j^{-1}(b_i))}{b_j'(k_j)}\left(-\frac{b_j''(k_j)}{\left(b_j'(k_j)\right)^2}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'(k_j)}\frac{g'(b_j^{-1}(b_i))}{g(b_j^{-1}(b_i))}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)\right.$$
$$\left.+(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'(k_j)}(b_i-c)-(\theta-b_j^{-1}(b_i))\right).$$

Once we substitute the candidate equilibrium $b_i(k)=b_j(k)$ the previous expression becomes

$$\frac{\partial^2\pi_i}{\partial^2 b_i(k_i)}=\frac{g(k_i)}{b^{*\prime}(k_i)}\frac{1}{a(k_i)}<0.$$

Because there is a unique solution to the first order condition, this implies that the profit function is quasiconcave and guarantees the existence of the equilibrium. In particular, this rules out deviations where firms choose any lower bid including c. □

Proof of Proposition 3: We want to show that there exists an equilibrium in which firm i behaves as in Proposition 2 for $k_i<\theta$, while it bids at marginal cost and offers $q_i=\lim_{\varepsilon\to 0}(\theta-\varepsilon)$ for all $k_i\geq\theta$. We only need to rule our deviations for $k_i\geq\theta$, as Proposition 2 already shows that deviations for $k_i<\theta$ are unprofitable. Suppose $k_i\geq\theta$, $q_i=\lim_{\varepsilon\to 0}(\theta-\varepsilon)$ and $b_i(k_i)=b_i\geq c$. At this candidate equilibrium, profits are

$$\pi_i(b_i,b_j(k_j)|k_i>\theta)=\int_{\underline{k}}^{b_j^{-1}(b_i)}(b_j(k_j)-c)\theta g(k_j)dk_j+\int_{b_j^{-1}(b_i)}^{\overline{k}}(b_i-c)(\theta-k_j)g(k_j)dk_j. \quad (10)$$

Since profits are increasing in q_i for $k_i<\theta$ (Lemma 1) we do not need to check deviations to $q_i<\theta$. If the firm deviates to $q_i>\theta$, for any candidate price offer $b_i\geq c$, profits are

$$\pi_i(b_i,b_j(k_j)|k_i>\theta)=\int_{\underline{k}}^{b_j^{-1}(b_i)}(b_i-c)\theta g(k_j)dk_j+\int_{b_j^{-1}(b_i)}^{\overline{k}}(b_i-c)(\theta-k_j)g(k_j)dk_j.$$

Comparing the above equation with (10), shows that the deviation is unprofitable: the second term is the same as in equation (10), while the first term is now smaller since, over this range, $b_j(k_j)>b_i$.

Thus, it only remains to show that the firm does not want to choose a bid above c. We show this by proving that profits are decreasing in $b_i>c$, as this implies that profits are are maximum at c. Taking derivatives,

$$\frac{\partial\pi_i}{\partial b_i}=b_j^{-1\prime}(b_i)g(b_j^{-1}(b_i))(b_i-c)b_j^{-1}(b_i)+\int_{b_j^{-1}(b_i)}^{\overline{k}}(\theta-k_j)g(k_j)dk_j. \quad (11)$$

From Proposition 2, using the equilibrium price offer for firm j,

$$b^*(k_i) = c + (P-c)\exp\left(-\omega(k_i)\right),$$

we can write

$$\exp\left(-\omega(k_j)\right) = \exp\left(-\omega(b_j^{-1}(b_i))\right) = \frac{b_i - c}{P-c}$$

Using the implicit function Theorem,

$$\frac{dk_j}{db_i} = -\frac{\int_{k_j}^{\overline{k}}(\theta-k)g(k)dk}{(b_i-c)\left(2k_j-\theta\right)g(k_j)}$$

Plugging this into the first-order condition 11, using $\frac{dk_j}{db_i} = b_j^{-1\prime}(b_i)$ and $b_j^{-1}(b_i) = k_j$, and simplifying

$$\begin{aligned}\frac{\partial \pi_i}{\partial b_i} &= \left(-\frac{k_j}{(2k_j-\theta)}+1\right)\left(\int_{k_j}^{\overline{k}}(\theta-k_j)g(k_j)dk_j\right)\\ &= \left(\frac{k_j-\theta}{2k_j-\theta}\right)\int_{k_j}^{\overline{k}}(\theta-k_j)g(k_j)dk_j < 0,\end{aligned}$$

as desired. □

Proof of Lemma 2: See Fabra et al. (2006). Unlike their paper, the fact that capacities are random and observable allows to symmetrize the equilibrium through perfect correlation between the two asymmetric pure strategy equilibria. □

Proof of Lemma 3: See Fabra et al. (2006). The proof is analogous to the case in which demand is uncertain (long-lived bids). □

Proof of Proposition 4: It follows from the proofs of Lemmas 2 and 3. □

Proof of Proposition 5: Profits for firm i are:

$$\pi_i(b_i, b_j|k_i) = \int_{\underline{k}}^{b_j^{-1}(b_i)}(b_j(k_j)-c)k_i\varphi(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}}(b_i-c)\left(\theta-k_j-\int_{k_j}^{\overline{k}}(N-2)\,kg(k)dk\right)\varphi(k_j)dk_j$$

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\begin{aligned}\frac{\partial \pi_i}{\partial b_i(k_i)} &= b_j^{-1\prime}(b_i)\varphi(b_j^{-1}(b_i))(b_i-c)\left(k_j+\int_{k_j}^{\overline{k}}(N-2)\,kg(k)dk + b_j^{-1}(b_i)-\theta\right)\\ &\quad+\int_{b_j^{-1}(b_i)}^{\overline{k}}\left(\theta-k_j-\int_{k_j}^{\overline{k}}(N-2)\,kg(k)dk\right)\varphi(k_j)dk_j = 0.\end{aligned}$$

Under symmetry, $b_j(k) = b_i(k)$, we can rewrite the expression as

$$\frac{\partial \pi_i}{\partial b_i(k_i)} = \frac{1}{b_i'(k_i)}\varphi(k_i)(b_i(k_i) - c)\left(2k_i + \int_{k_i}^{\overline{k}} (N-2)\, kg(k)dk - \theta\right) + \int_{k_i}^{\overline{k}} \left(\theta - k_j - \int_{k_j}^{\overline{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j = 0$$

Reorganizing it,

$$b_i'(k_i) + b_i(k_i)a(k_i) = ca(k_i)$$

where $a(k_i)$ does not depend on b_i,

$$a(k_i) = \frac{\left(2k_i + \int_{k_i}^{\overline{k}} (N-2)\, kg(k)dk - \theta\right)\varphi(k_i)}{\int_{k_i}^{\overline{k}} \left(\theta - k_j - \int_{k_j}^{\overline{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j}.$$

Hence, the solution is the same as above:

$$b_i^*(k_i) = c + (P - c)\, e^{-\omega(k_i)}$$

where $\omega(k_i) \equiv \int_{\underline{k}}^{k_i} a(k)dk$. □

Proof of Proposition 6: We show that there is no profitable deviation from the candidate equilibrium stated in the text of the proposition.

Regarding part (i), the same logic as in Proposition 1 applies.

For part (ii), let's start by focusing on $k_i \in [\underline{k}_1, \overline{k}_2]$. The profit function of both firms can be written as

$$\pi_i(b_i, b_j(k_j)|k_i \in [\underline{k}_1, \overline{k}_2]) = (P - c)k_i G_i(\hat{\underline{k}}_1) + \int_{\hat{\underline{k}}_1}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g_j(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\overline{k}_2} (b_i - c)(\theta - k_j)g_j(k_j)dk_j.$$

Under the assumption that $g_i(k_i)$ is uniformly distributed in an interval of the same length, we have $g_i(k_i) = g_j(k_j)$ for $k_i \in [\underline{k}_i, \overline{k}_i]$ and $i = 1, 2$. As a result, the profit function of the two firms is identical because the bid function in this range is the same. Hence, the first condition is also the same and it coincides with equation (7) in the proof of Proposition 1, leading to expression (3).

Using arguments similar to those in previous propositions, it can be shown that $b_2(k_2) = P$ for $k_2 < \underline{k}_1$ and $b_1(k_1) = c$ when $k_1 > \overline{k}_2$. □

Proof of Lemma 4: We first prove the non-existence of a pure-strategy equilibrium when $k_i \geq \theta$. By way of contradiction, assume that there exists one. Following the same steps as in the proof of Lemma 1, it is easy to show that it must be non-increasing in k_i. Suppose, therefore, that b_j is non-increasing in k_j. As a result, the optimal bid for firm i can be characterized as $b_i \in \arg\max_{b_i} \pi_i(b_i, b_j(k_j)|k_i)$.

If $b_i > b_j(\theta)$, expected profits are

$$\pi_i(b_i, b_j(k_j)|k_i) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j)g(k_j)dk_j\right).$$

Instead, if $b_i \leq b_j(\theta)$,

$$\pi_i(b_i, b_j(k_j)|k_i) = (b_i - c)\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j)dk_j.$$

In both cases, profit functions do not depend on k_i. Therefore, the optimal bid is the same for all $k_i \geq \theta$. Thus, at the candidate pure strategy equilibrium, $b^*(k_i) = b^*(\theta)$ for all $k_i \geq \theta$.

However, this is ruled out by standard Bertrand-Edgeworth arguments. First, if $b^*(\theta) > c$, firm i would have incentives to slightly undercut $b^*(\theta)$. If $k_j \geq \theta$, this would allow firm i to serve total demand, rather than a share of it, at only a slightly lower price, with almost no effect on firm i's profits if $k_j < \theta$. Second, if $b^*(\theta) = c$, the market price would always be c. Hence, firm i would make zero profits regardless of k_j and would rather deviate to P in order to make positive profits over the expected residual demand. It follows that the equilibrium must involve mixed strategies. Standard arguments imply that firms choose prices in a compact support $\left[\underline{b}, \bar{b}\right]$. □

Proof of Proposition 7: A symmetric Bayesian Nash Equilibrium must have the following properties. First, using the same arguments in Proposition 2, the optimal bid must be strictly decreasing in k_i for $k_i < \theta$. Second, from Lemma 4 it must involve a mixed strategy when $k_i \geq \theta$.

To make things simpler, we first assume $\bar{b} \leq b(\underline{k})$ and $\underline{b} \geq b(\theta)$. At the end of the proof we will show that this assumption must hold in equilibrium. We define $\tilde{k} = b_j^{-1}(\bar{b})$ and $\hat{k} = b_j^{-1}(\underline{b})$. Since $b_j(k_j)$ is decreasing, it follows that $\left[\tilde{k}, \hat{k}\right] \subseteq [\underline{k}, \theta]$. We consider four capacity regions:

Region I. If $k_i \in \left[\underline{k}, \tilde{k}\right]$, expected profits are

$$\pi_i(b_i, b_j(k_j)|k_i) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j)g(k_j)dk_j$$

Firm i has the low bid when $k_j < b_j^{-1}(b_i)$ and, hence sells up to capacity at the price set by firm j. Otherwise, either it sells the residual demand and sets the price or, if $k_j > theta$ the rival will serve all the market.

Taking derivatives, we obtain a similar First Order Condition as in equation (7), with the only difference that $\overline{k}$ is replaced by θ. Hence, the solution is the same as in Proposition 2, with the only difference that $\overline{k}$ is replaced by θ in equation (8). Hence, the optimal bid in this region is

$$b^*(k_i) = c + (P - c)\, e^{-\omega(k_i)}, \tag{12}$$

where $\omega(k_i) \equiv \int_{\underline{k}}^{k_i} a(k)dk$, and

$$a(k) = \frac{(2k - \theta)g(k)}{\int_{\underline{k}}^{\theta}(\theta - k_j)g(k_j)dk_j}. \tag{13}$$

Using the optimal bid in (12), for given $\bar{b}$, $\tilde{k}$ is implicitly defined by

$$b^*\left(\tilde{k}\right) = \bar{b}.$$

Region II. If $k_i \in \left[\tilde{k}, \hat{k}\right]$, expected profits are

$$\begin{aligned}\pi_i(b_i, b_j(k_j)|k_i) \quad &= \quad \int_{\underline{k}}^{b_j^{-1}(b_i)}(b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta}(b_i - c)(\theta - k_j)g(k_j)dk_j \\ &\quad + (1 - G(\theta)) \int_{b_i}^{\bar{b}}(b_j - c)k_i f_j(b_j)\, db_j.\end{aligned}$$

The profit expression now adds a third term as the firm will serve all its capacity at the price set by the rival whenever $k_j \geq \theta$ and $b_i < b_j$.

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{1}{b_j'(k_j)}g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\theta}(\theta - k_j)g(k_j)dk_j - (1 - G(\theta))(b_i - c)k_i f_j(b_i) = 0.$$

This expression is similar to equation (7), where $\overline{k}$ replaces θ, plus an additional third term, which is negative. If follows that the optimal bid that solves the above equation is lower than the optimal bid in the baseline case.

Using symmetry, the optimal bid is the solution to

$$\left(1 - \frac{(1 - G(\theta))(b(k) - c)kf(b(k))}{(2k - \theta)g(k)}a(k)\right) b'(k) + a(k)b(k) = ca(k),$$

where $a(k)$ is defined as in equation (13). Note that if $G(\theta) = 1$ we would obtain the same solution as in the baseline case. Since we now have $G(\theta) < 1$, the solution is lower.

Region III. If $k_i \in \left[\hat{k}, \theta\right]$, expected profits are

$$\begin{aligned}\pi_i(b_i, b_j(k_j)|k_i) &= \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j)g(k_j)dk_j \\ &\quad + (1 - G(\theta)) \int_{\underline{b}}^{\bar{b}} (b_j - c)k_i f_j(b_j)\, db_j.\end{aligned}$$

The first-order condition that characterizes the optimal bid of firm i is the same as in Region I as the last term does not depend on b_i. Hence, the solution is also given by expressions (12) and (13). Hence, (12), for given $\underline{b}$, $\hat{k}$ is implicitly defined by $b^*(\hat{k}, \underline{k}, \theta) = \underline{b}$.

Region IV. Last, consider $k_i \in \left[\theta, \overline{k}\right]$. Expected profits are given by,

$$\pi_i(b_i, b_j(k_j)|k_i) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j)g(k_j)dk_j + (1 - F_j(b_i))(1 - G(\theta))\theta\right) \tag{14}$$

As argued above, this profit function does not depend on k_i, so the optimal bid must be constant in k_i.

At the upper bound of the support, $F_j\left(\bar{b}\right) = 1$. Hence, $\bar{b}$ maximizes

$$\begin{aligned}\pi_i(\bar{b}, b_j|k_i) &= (\bar{b} - c)\left(\int_{\underline{k}}^{b_j^{-1}(\bar{b})} \theta g(k_j)dk_j + \int_{b_j^{-1}(\bar{b})}^{\theta} (\theta - k_j)g(k_j)dk_j\right) \\ &= (b^*\left(\tilde{k}\right) - c)\left(\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j)dk_j\right)\end{aligned}$$

Taking derivatives with respect to $\bar{b}$,

$$\theta G(\theta) - \int_{b_j^{-1}(\bar{b})}^{\theta} k_j g(k_j)dk_j + (\bar{b} - c)\frac{1}{b_j'(k_j)} g(b_j^{-1}(\bar{b}))b_j^{-1}(\bar{b}) = 0$$

Using the definition of $\tilde{k}$ above, it can be re-written as

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j)dk_j + (\bar{b} - c)\frac{1}{b_j^{*\prime}(\tilde{k})} g(\tilde{k})\tilde{k} = 0.$$

From the analysis of the case with small capacities we know that

$$b_i'(k_i) + a(k_i)b_i(k_i) = ca(k_i)$$

so that

$$b_j'(k_j) = -\left(b_i(k_i) - c\right)a(k_i)$$

Hence,

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j - (\bar{b} - c) \frac{1}{\left(b^*(\tilde{k}) - c\right) a(\tilde{k})} g(\tilde{k})\tilde{k}$$

Since $b^*(\tilde{k}) = \bar{b}$,

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j - \frac{g(\tilde{k})\tilde{k}}{a(\tilde{k})} = 0$$

Using the expression for $a(k)$ in equation (13),

$$\theta G\left(\tilde{k}\right) - \frac{\theta - \tilde{k}}{2\tilde{k} - \theta} \int_{\tilde{k}}^{\theta} (\theta - k_j)\, g(k_j) dk_j = 0,$$

which defines $\tilde{k}$. Note that we must have an interior solution, $\tilde{k} \in (\underline{k}, \theta)$. For $\tilde{k} = \underline{k}$, the first term is zero so the left hand side would be negative; whereas for $\tilde{k} = \theta$, the second term is zero so the left hand side would be positive.

At the lower bound of the support, $F_j(\underline{b}) = 1$. Expected profits are

$$\begin{aligned} \pi_i(\underline{b}, b_k | k_i) &= (\underline{b} - c)\left(\theta - \int_{b_j^{-1}(\underline{b})}^{\theta} k_j g(k_j) dk_j\right) \\ &= \left(b\left(\hat{k}\right) - c\right)\left(\theta - \int_{\hat{k}}^{\theta} k_j g(k_j) dk_j\right) \end{aligned}$$

Since the firm must be indifferent between all the prices in the support, profits at the lower and upper bounds must be equal,

$$(\bar{b} - c)\left(\theta G(\theta) - \int_{b_j^{-1}(\bar{b})}^{\theta} k_j g(k_j) dk_j\right) = (\underline{b} - c)\left(\theta - \int_{b_j^{-1}(\underline{b})}^{\theta} k_j g(k_j) dk_j\right) = \pi^*$$

Using the definitions for $\bar{b}$ and $\underline{b}$,

$$\left(b^*\left(\tilde{k}\right) - c\right)\left(\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j\right) = \left(b^*\left(\hat{k}\right) - c\right)\left(\theta - \int_{\hat{k}}^{\theta} k_j g(k_j) dk_j\right) = \pi^*$$

which defines $\hat{k}$. Hence, equilibrium profits are well defined $\underline{\pi}$ and we can treat them like a constant.

By the above equality, when $\overline{k}$ is just above θ, $\tilde{k}$ is arbitrarily close to $\hat{k}$. Instead, when $\underline{k}$ is so large that $G(\theta) = 0$, then $b\left(\hat{k}\right) = c$.

Now, we can use the above expression for equilibrium profits to solve for $F(b)$ in equation (14),

$$F(b) = \frac{1}{(1 - G(\theta))\,\theta}\left(\theta - \int_{b^{*-1}(b)}^{\theta} k g(k) dk - \frac{\pi^*}{(b - c)}\right)$$

where $b^*(k)$ is defined above by expressions (12) and (13).

Computing the density,

$$f(b) = \frac{1}{(1 - G(\theta))\,\theta} \left(\frac{\pi^*}{(b-c)^2} + \frac{k}{b^{*\prime}(k)} g(k) \right).$$

$\square$