Product Choice and Price Discrimination by Competing Firms*

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Abstract

In a seminal paper, Champsaur and Rochet (1989) showed that competing firms choose non-overlapping qualities so as to soften price competition at the cost of giving up price discrimination. Does this equilibrium survive under imperfect competition? In this paper we show that the source of market power matters. Whereas the equilibrium with non-overlapping qualities survives the introduction of a small amount of horizontal differentiation, it vanishes in the presence of even an arbitrarily small amount of search frictions. Instead, in markets with search frictions, there always exist an equilibrium with overlapping qualities and full price discrimination. These results have implications for the relationship between competition and price discrimination, and the resulting equilibrium product choices and prices.

Keywords: second degree price discrimination, search, product differentiation, vertical differentiation, retail competition.

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1 Introduction

Since the classical work of Chamberlin (1933), a well known principle in economics is that firms differentiate their products in order to relax competition. Champsaur and Rochet (1989) (CR, thereafter) formalized this Chamberlinian incentive in a model in which quality choices are followed by price competition.[1] They showed that firms choose non-overlapping product lines because their incentives to soften price competition dominate over their incentives to price discriminate consumers according to their quality preferences. Yet, in many markets, competing firms often carry overlapping qualities, even when this creates fierce competition among them. How can this fact be reconciled with CR's prediction?

Imperfect competition could be a reason. As already hypothesized by CR, if competition is not strong enough, the gains from price discrimination would dominate over the incentives to soften competition, inducing firms to carry wider product lines (see CR, p. 535). This was confirmed by Gal-Or (1983) who showed that, in a Cournot setting, firms' product lines overlap in equilibrium. However, moving from CR's Bertrand assumption to Gal-Or's (1983) Cournot assumption represents a discrete jump in market power: as is well known, Bertrand and Cournot represent two extremes of a continuum of cases. However, in real-world markets, firms' supply functions are rarely perfectly elastic or perfectly inelastic, as the two models respectively assume.

In this paper we investigate the consequences of introducing imperfect competition in CR's pricing game, albeit not as imperfect as in the Cournot model. In particular, we want to understand whether in markets characterized by a small degree of market power, firms would still prioritize the softening of competition effect over the gains from price discrimination. This issue has important consequences for equilibrium market outcomes, including product choices and prices.

Two sources of imperfect competition immediately come to mind: horizontal differentiation *à la* Hotelling (1929) and search costs *à la* Varian (1980). Needless to say, there are other sources of market power. However, focusing on these two already allows us to highlight one important conclusion: the source of imperfect competition matters for the relationship between competition and price discrimination. Indeed, as we show in this paper, CR's equilibrium with non-overlapping qualities survives the introduction of small amounts of horizontal differentiation, but fails to survive in the presence of even an arbitrarily small mass of non-shoppers (i.e., consumers who never search because of

[1] Shaked and Sutton (1982) formalized the same idea in a model similar to Champsaur and Rochet's (1989), with the difference that firms are allowed to offer one quality only. Thus, in Shaked and Sutton (1982), there is no possibility to discriminate consumers at the firm level.

high search costs).[2]

To understand why, consider CR's equilibrium in which one firm offers low qualities and the other firm offers high qualities, with a gap in between. In markets with search frictions, no matter how small, the low-quality firm would always find it worthwhile to deviate by carrying very high qualities, above the ones offered in CR's equilibrium by the high-quality firm. This deviation is profitable as it would allow the firm to better discriminate the non-shopper high types (i.e., those with high quality preferences), without affecting the competition for the shoppers (i.e., consumers who search for the lowest price at no cost). Furthermore, the firm could do even better than this: by carrying qualities that are not as high, the firm could both attract some shoppers and, more importantly, extract more rents by better discriminating the non-shoppers. At the margin, this latter gain dominates any eventual loss due to more intense competition for the shoppers. Anticipating that competition will be stronger, the high-quality firm has incentives to close the original gap, with the consequent unraveling of CR's equilibrium.

In contrast, in the presence of small amounts of horizontal differentiation, firms do not want to deviate from CR's equilibrium. The low-quality firm faces the same trade-off as with search frictions: deviating to carry high qualities outside the gap helps her to better discriminate the high-types at the cost of intensifying competition with the rival's high quality goods, and hence, at the cost of depressing the price of low qualities. There is a big difference, however: under search frictions, there are deviations that allow the low-quality firm to increase its profits from the captive consumers, without affecting competition whatsoever; in contrast, under horizontal differentiation, any deviation outside the gap entails, by construction, stealing (high-type) consumers from the rival. In the latter case, the rival would react with deep –as opposed to marginal– price cuts, making such a deviation unprofitable. This is exactly the opposite to that occurs under search frictions.[3]

Given that CR's non-overlapping equilibrium fails to exist in markets with search frictions, we are interested in characterizing the new equilibrium. In stark contrast with CR's, search frictions give rise to an equilibrium with fully overlapping qualities, even when such equilibrium results in low profits. When search is costly, the marginal in-

[2]This conclusion remains valid regardless of the distribution of the search costs of the remaining buyers, a long as there is a positive mass of non-shoppers. It is also valid regardless of whether the non-shoppers visit one firm at random, or whether they visit the one that gives them higher ex-ante utility.

[3]Analyzing a completely different issue (the impact of exclusive dealing contracts on market foreclosure), Simpson and Wickelgren (2007) also highlight that the way to introduce frictions to an otherwise Bertrand market may matter a great deal for equilibrium formation. In their case, the no-foreclosure equilibrium that emerges under Bertrand competition would survive the introduction of arbitrarily small amounts of search frictions but it would not to the introduction of equally small amounts of horizontal differentiation.

centives faced by firms mimic those of a monopolist: firms' incentives to discriminate consumers through quality choices dominate over their incentives to soften price competition. This induces firms to offer the full quality range in order to implement the monopoly solution.

Therefore, the comparative statics of equilibrium outcomes with respect to horizontal differentiation and with respect to search frictions are very different. Essentially, search frictions affect quality choices (i.e., whether product lines overlap or not), and through that, they end up affecting prices, qualities and consumer surplus. There are two effects at play: on the one hand, an arbitrarily small amount of search frictions intensifies competition and increases product variety by giving rise to overlapping quality choices; on the other, further increases in search frictions relax competition, eventually leading to prices above those in frictionless markets. In contrast, introducing a small amount of horizontal differentiation smoothly enlarges product variety, which in turn reduces prices, as firms manage to avoid any product overlap.

Last, in building the equilibrium with non-overlapping qualities, we generalize CR to settings in which there exist consumers with low reservation prices (CR implicitly assume that even the lowest type has a sufficiently high reservation price). The solution gives rise to new equilibrium patterns, even if CR's qualitative prediction remains unchanged –namely, that in the absence of search costs, firms can credibly relax competition by carrying non-overlapping qualities.

Related Literature.– We contribute to a couple of strands in the literature. First, we add to the growing literature that analyzes competition with search costs.[4] Since the seminal work of Diamond (1971), the search literature has shown that the introduction of search frictions can have substantial effects on competition, no matter how search is modeled.[5] However, unlike CR, this literature has broadly neglected the possibility that firms engage in price discrimination through quality choices.[6] The vast part of the

[4]There is also a large empirical literature investigating price discrimination in markets where search costs matter, with a focus on price patterns. There are studies on gasoline markets, where consumers have the choice of paying for full-service or self-service gasoline at the same station, or of searching for competing stations (Shepard, 1991); the airline industry, where travellers can choose whether to fly in business or in economy class, or just in economy class but with certain restrictions (Borenstein and Rose, 1994; Gerardi and Shapiro, 2009); coffee shops (McManus, 2000), cereals (Nevo and Wolfram, 2002), theaters (Leslie, 2004), Yellow Pages advertising (Busse and Rysman, 2005), and cable TV (Crawford and Shum, 2007), among others.

[5]Search models can essentially be classified as models of either simultaneous search (Burdett and Judd, 1983) or sequential search (Stahl, 1989). De los Santos *et al.* (2012) test which of the two processes best represents actual search for online books, and conclude in favor of the simultaneous search model, which is the approach we adopt in this paper.

[6]Unlike the current paper, in which we model second-degree price discrimination, Fabra and Reguant

search literature assumes that consumers search for one unit of an homogenous good, with two exceptions. Some search models allow for product differentiation across firms but, unlike ours, assume that each firm carries a single product.[7] Other search models allow firms to carry several products but, unlike ours, typically assume that consumers search for more than one ('multi-product search').[8] In these models, consumers differ in their preference for buying all goods in the same store ('one-stop shopping') rather than on their preferences for quality.[9] These differences are relevant. In the first type of search models, the single-product firm assumption leaves no scope for price discrimination within the firm. Hence, pricing is solely driven by competitive forces. In the second type of search models, the multi-product search assumption implies that discrimination is based on heterogeneity in consumers' shopping costs, which become the main determinant of firms' product choices (Klemperer, 1992).

Within the 'multi-product search' literature, two papers deserve special attention. In line with our results, Zhou (2014) finds that multi-product firms tend to charge lower prices than single-product firms. This is not driven by the interaction between competition and price discrimination, as in our paper, but rather by a 'joint search' effect, i.e., multi-product firms charge less because they gain more by discouraging consumers from searching competitors. In Rhodes and Zhou (2019), increases in search costs imply that consumers value one-stop shopping more, thus making it more likely that the equilibrium involves multi-product firms. For small search costs, Rhodes and Zhou (2019) predict asymmetric market structures with single-product and multi-product firms coexisting. The driving force underlying our predictions is quite different: since in our model consumers buy a single good, the multi-product firm equilibrium is not driven by one-stop shopping considerations but rather by firms' incentives to price discriminate consumers with heterogenous quality preferences. Despite these differences, our paper has one common prediction with both Rhodes (2014) and Rhodes and Zhou (2019): namely, search frictions can give rise to lower prices through their effect on endogenous product choices.

As far as we are aware of, Garret *et al.* (2019) is the only paper that, like ours,

(2019) allow for third-degree price discrimination in markets with search costs.

[7]For models with horizontal product differentiation, see for instance Anderson and Renault (1999) and Bar-Isaac *et al.* (2012); see Ershow (2017) for an empirical application. Wildenbeest (2011) allows for vertical differentiated products but, unlike us, assumes that all consumers have the same preference for quality; hence, there is no scope for price discrimination.

[8]There is a recent strand of papers in the ordered search literature that analyze obfuscation by multi-product firms (Gamp, 2016; Petrikaite, 2017). Their emphasis is on the monopoly case. See Armstrong (2017) for a discussion.

[9]One-stop shopping considerations are also the driving force behind the evidence of price dispersion across stores documented by Kaplan *et al.* (2019).

introduces search frictions in a model of price discrimination.[10] Given our focus on how market frictions, whether from search or horizontal differentiation, affect firms' ability to commit to asymmetric product lines, we build our analysis on a two-stage game –first, firms choose which qualities to carry and then, they decide how to price those qualities–. Instead, Garret *et al.* (2019) characterize the symmetric equilibrium of a one-stage game in which firms are unconstrained in the qualities they can offer.

Our paper also relates to the literature that analyzes quality choices followed by imperfect competition, either quantity competition (Gal-Or, 1983; Wernerfelt, 1986; Johnson and Myatt 2003, 2006 and 2015) or price competition with horizontal differentiation (Gilbert and Matutes, 1993; Stole, 1995). As already noted by CR (p. 535), one of the main consequences of less competitive pricing is to induce wider and, very likely, overlapping product lines. While one may view search frictions as equivalent to other forms of imperfect competition, our paper shows they are not.

The rest of the paper is organized as follows. Section 2 describes the model, and reviews the competitive and monopoly solutions. Section 3 revisits CR's non-overlapping equilibrium in the absence of frictions. Section 4 introduces horizontal differentiation and search frictions, and shows that the non-overlapping equilibrium survives the former but not the latter. Section 5 characterizes the new equilibrium under search frictions, and shows that it involves full quality overlap. Section 6 of the paper concludes, and all proofs are postponed to the Appendix.

2 Baseline Model

Consider a market served by two firms that carry a set of qualities Q_i in $\mathbb{R}^+$, $i = 1, 2$. A firm's product line Q_i can include qualities within an interval, or within a finite number of disjoint intervals.[11] The cost of a particular quality $q \in Q_i$, denoted $C(q)$, is assumed to be strictly increasing and convex, with $C(0) = C'(0) = 0$. There is a unit mass of consumers who buy at most one good. Consumers differ in their preference for quality,

[10] Another set of related papers analyze pricing for add-ons. Ellison (2005) and Verboven (1999) consider models in which consumers are well informed about base product prices but don't know the price of the add-ons, unless they search. Critically, in these models the customers that are more likely to buy the add-ons are also less likely to search. Our search model is not a model of add-on pricing because shoppers observe all prices and non-shoppers only those of the firm they visit, and this applies symmetrically for both products regardless of their quality. Furthermore, our results hold regardless of whether there is correlation or not between consumers' quality preferences and search cost types. See the discussion in Section 5.2.

[11] In CR, product lines are constrained to be an interval. We will consider equilibria in which product lines are an interval too, but unlike CR, we will consider potential deviations outside the interval.

as captured by their type θ. Types are drawn from a continuous distribution $F(\theta)$ with density $f(\theta) > 0$ in a closed interval $\left[0, \bar{\theta}\right]$.[12] Following Mussa and Rosen (1978) (MR, thereafter), the utility of type θ buying quality q at a price $p(q)$ is given by

$$U(\theta) = \theta q - p(q),$$

while the utility of not buying a good is normalized to zero.

For tractability purposes, we provide closed form solutions for MR's and CR's leading specification, which has quadratic costs, $C(q) = q^2/2$, and uniformly distributed types in $\left[0, \bar{\theta}\right]$ (*quadratic-uniform* case).

We consider a two-stage game with the following timing. First, simultaneously and independently, firms choose their product lines Q_i, $i = 1, 2$. Once chosen, (Q_1, Q_2) become observable to firms and to consumers, unless we indicate otherwise. Second, firms post menus of contracts with different quality-price combinations, under the constraint that all the qualities offered by firm $i = 1, 2$ must be contained in its product line Q_i. Last, consumers buy from the firm that offers them higher utility.

2.1 Perfect competition and Monopoly reviewed

For future reference, it is useful to review the solutions under perfect competition and monopoly. At the competitive solution, the consumer's marginal utility equals marginal cost at his optimal quality choice, $\theta = C'(q)$. In the quadratic-uniform setting, quality at the competitive menu is $q_c(\theta) = \theta$. Furthermore, since consumers extract all the surplus, $U_c(\theta) = \theta^2/2$.

A monopolist, on the other hand, chooses a set of menus $\{q(\theta), p(\theta)\}$ to maximize profits

$$\pi = \int \left[p(\theta) - C(q(\theta))\right] f(\theta)\, d\theta,$$

subject to the incentive compatibility constraints, $U(\theta) = \theta q(\theta) - p(\theta) \geq \theta q(\theta') - p(\theta')$ for all $\{\theta, \theta'\} \in \left[0, \bar{\theta}\right]$; and subject to the participation constraints, $U(\theta) \geq 0$ for all $\theta \in \left[0, \bar{\theta}\right]$.

Using the Envelope Theorem, the optimality condition $U'(\theta) = q(\theta)$ implies that each type must obtain utility

$$U(\theta) = U(\theta^*) + \int_{\theta^*}^{\theta} q(s)\, ds, \tag{1}$$

[12] Setting $\underline{\theta} = 0$ reduces the number of cases we need to consider, while at the same time it gives rise to new results that do not appear in CR. Even though they do not mention it explicitly, their results apply only to the case in which $\underline{\theta}$ is sufficiently large. This explains why some of the results we derive in Section 3 do not fully coincide with those in CR, even though the qualitative nature of the two remains unchanged.

where θ^* is the lowest type being served. Since it is optimal to set $U(\theta^*) = 0$, prices can then be written as

$$p(\theta) = \theta q(\theta) - U(\theta) = \theta q(\theta) - \int_{\theta^*}^{\theta} q(s)\, ds.$$

Solving the monopoly problem, the optimal quality for type θ is characterized by

$$C'(q_m(\theta)) = \theta - \frac{1 - F(\theta)}{f(\theta)},$$

and the lowest type being served θ^* is given by

$$C(q_m(\theta^*)) = \left[\theta^* - \frac{1 - F(\theta^*)}{f(\theta^*)}\right] q_m(\theta^*).$$

As is well known, there is a downward distortion of quality for all types except for the highest one, and not all types are served.

In the quadratic-uniform case, the quality schedule is given by $q_m(\theta) = 2\theta - \bar{\theta}$ if $\theta \in [\bar{\theta}/2, \bar{\theta}]$ and zero otherwise, i.e., $\theta^* = \bar{\theta}/2$. Utilities are $U_m(\theta) = (\theta - \bar{\theta}/2)^2$ if $\theta \in [\bar{\theta}/2, \bar{\theta}]$ and zero otherwise. Last, monopoly profits are $\pi_m = \bar{\theta}^2/12$. This is the MR solution to which we will frequently refer throughout the paper.

3 Equilibrium in a Frictionless Market

Our point of departure is CR's equilibrium. In a frictionless market in which firms compete in prices, CR show that firms give up opportunities to price discriminate in order to relax competition. In particular, firms avoid any quality overlap - which would lead to prices equal to marginal costs for such qualities - and further relax competition by leaving a gap between the two firms' product lines. In particular, in the absence of frictions, CR's equilibrium takes the following form:[13]

Proposition 1 *Consider the quadratic-uniform case. In a frictionless market, the pair of product lines* $Q_1 = [0, q_1^+]$ *and* $Q_2 = [\bar{\theta}, \infty)$, *with* $q_1^+ = q_m(\theta = q_1^+) = (2 - \sqrt{2})\bar{\theta}$, *constitutes a subgame perfect Nash equilibrium (SPNE) of the two-stage game.*

Proof. See the Appendix. ■

[13] As mentioned, we depart from CR on an important assumption: whereas they assume that a "consumer always buys something", thus implicitly assuming an infinite reservation price, in our model the participation constraint need not be satisfied for all types. This key difference explains why the results in this Proposition differ from those in CR's Proposition 3, in which each firm produces a unique quality at the extremes.

In equilibrium, firm 1 and 2 offer a range of low and high quality products, respectively, with a gap in between, i.e., $q_1^+ < q_2^-$. Firm 1 discriminates consumer types up to $\theta \leq q_1^+$, from whom it obtains monopoly profits, and sells quality q_1^+ to consumers $\theta \in \left[q_1^+, \tilde{\theta}\right]$, where type $\tilde{\theta}$ is indifferent between buying q_1^+ at $p_1\left(q_1^+\right)$ and $q_2^- = \bar{\theta}$ at $p_1\left(q_2^-\right)$. Firm 2 sells a single quality $q_2^- = \bar{\theta}$ to the remaining consumers. Qualities above $\bar{\theta}$ are not bought in equilibrium, but they play a strategic role as they discourage firm 1 from offering those qualities (as any overlap would lead to Bertrand pricing). Thus, consumers $\theta \leq \tilde{\theta}$ buy inefficiently low qualities from firm 1, except for $\theta = q_1^+$, while consumers $\theta \geq \tilde{\theta}$ buy inefficiently high qualities from firm 2, except for $\bar{\theta} = q_2^-$.

As explained by CR, firms do not want to expand their product lines: whereas this would allow firms to better discriminate consumers in the gap (i.e., those who buy either q_1^+ or q_2^-), it would also intensify competition among them, leading to lower prices for all consumer types. There is however an important distinction between our equilibrium and the one characterized in CR's Proposition 4. Unlike CR, the schedule offered in equilibrium by firm 1 to consumers $\theta \leq q_1^+$ is not affected at all by firm 2's offer; it is a MR type of schedule, with no distortion at the top of its quality range, $q_m(q_1^+) = q_1^+$.

Several forces contribute to this result. As in CR, both firms have incentives to keep q_1^+ and q_2^- apart. Firm 2 does so, not only to soften competition for consumers in the gap, but also to reduce the outside option of consumers upon whom it exerts local monopoly power (those who consume qualities above q_2^-). In the quadratic-uniform setting, these two forces push firm 2 all the way up to $q_2^- = \bar{\theta}$. Similarly, when q_1^+ and q_2^- are close enough, firm 1 also wants to reduce q_1^+ to both soften competition in the gap as well as to reduce the outside option of consumers upon whom it exerts local monopoly power (those who consume qualities below q_1^+). However, when q_1^+ and q_2^- are sufficiently apart, firm 1's problem changes radically. In firm 2's problem, q_1^+ is always a relevant outside option for firm 2's customers as higher types are always tempted to buy lower quality goods. But this is not always the case in firm 1's problem as lower types are not tempted to buy higher quality goods when q_2^- is sufficiently apart from q_1^+.

In fact, if q_1^+ drops below the equilibrium level in Proposition 1 while q_2^- stays unchanged, firm 1 faces a MR's monopoly problem because q_2^- is no longer a relevant outside option for firm 1's consumers. As soon as this happens, exercising full monopoly power upon these consumers dominates the gain from further softening competition for consumers in the gap. As a result, q_1^+ is not pushed further apart from q_2^-, but it instead remains at the corner where firm 1 can exercise maximum monopoly power upon its captive consumers. This is in contrast to CR's model, in which the implicit restriction that $\underline{\theta}$ cannot be too low stops q_1^+ from falling down enough so as to be sufficiently far from q_2^-. Therefore, the fundamental asymmetry in the incentives faced by the two firms never arises in CR. As a result, the price of q_1^+ in CR is determined *as if* each firm offered a single quality (see their Proposition 1), resulting in an equilibrium in which both firms offer a single quality at the two extremes of the quality range (see their Proposition 3). Despite this difference between the two models, the qualitative prediction remains the same: in a frictionless environment (i.e., where Bertrand price competition prevails), firms carry non-overlapping qualities in equilibrium.

4 Adding Market Frictions

Since markets are rarely frictionless, in this section we investigate whether CR's softening of competition prediction is robust to introducing imperfect competition, albeit not as imperfect as in the Cournot model (Gal-Or, 1983). We consider two types of market frictions: horizontal differentiation *à la* Hotelling (1929) and search costs *à la* Varian (1980). Beyond their relevance, the reason for focusing on these two is that they lead to contrasting outcomes, as we will see next.

4.1 Horizontal differentiation

Suppose that the two firms are located at the extremes of a Hotelling city of length 1. Consumers are uniformly distributed over the city, so that at each location $x \in [0,1]$ there is a unit mass of consumers with preferences $\theta \in \left[0, \bar{\theta}\right]$, drawn from the continuous distribution $F(\theta)$. Thus, the utility of type θ located at x who buys quality q at a price $p(q)$ from firm $i = 1, 2$ is given by

$$U(\theta, x) = \theta q - p(q) - t|x - x_i|$$

where t captures the degree of product differentiation between the two firms, located at $x_1 = 0$ and $x_2 = 1$.

Solving the game with $t > 0$ is conceptually similar but more involved than solving CR's game. The reason is that, for given prices and quality choices, the demand faced by each firm varies across locations. As in CR, consider a candidate equilibrium with non-overlapping qualities, $Q_i = \left[q_i^-, q_i^+\right]$, $i = 1, 2$, with $q_1^+ < q_2^-$. At each location x, there are two indifferent types: the one indifferent between not buying at all and buying from firm 1,

$$\theta'(x) = \frac{p_1^- + tx}{q_1^-},$$

where $p_1^- \equiv p_1(q_1^-)$, and the one indifferent between buying from firm 1 or from firm 2,

$$\tilde{\theta}(x) = \frac{p_2^- - p_1^+ + t(1-2x)}{q_2^- - q_1^+}$$

where $p_1^+ \equiv p_1(q_1^+)$ and $p_2^- \equiv p_2(q_2^-)$. The overall demand faced by firm 1 is thus given by the mass of consumers located between the two indifferent types, summed across all locations $x \in [0,1]$.

How does the equilibrium characterized in Proposition 1 change when we add an arbitrarily small amount of product differentiation, i.e., $t \to 0$? The next Proposition provides the answer.

Proposition 2 *Consider the quadratic-uniform case. If $t \to 0$, the pair of product lines $Q_1 = \left[0, q_1^+\right]$ and $Q_2 = \left[\bar{\theta}, \infty\right)$, with $q_1^+ \in ((2-\sqrt{2})\bar{\theta}, \bar{\theta})$ and solving*

$$\frac{3}{4}(q_1^+)^2 - t + \frac{t^2}{(q_1^+)^2} = \frac{1}{8\bar{\theta} - 2q_1^+}\left(3q_1^+\bar{\theta}^2 - 2t\bar{\theta} + 2q_1^+ t\right), \tag{2}$$

constitutes a subgame perfect Nash equilibrium (SPNE) of the two-stage game.

Proof. See the Appendix. ■

The equilibrium still looks very similar to the one in Proposition 1. In particular, firms choose non-overlapping qualities with a gap in between: firm 2 is pushed to the high quality end, $Q_2 = \left[\bar{\theta}, \infty\right)$, while firm 1 offers a low quality range, $Q_1 = \left[0, q_1^+\right]$, with $q_1^+ < \bar{\theta}$.

There are two main differences with respect to the equilibrium in Proposition 1. First, the introduction of $t > 0$ increases the equilibrium value of q_1^+ which is now closer to $q_2^- = \bar{\theta}$ (note that when $t = 0$, (2) yields $q_1^+ = (2-\sqrt{2})\bar{\theta}$, as in Proposition 1). Intuitively, an increase in horizontal differentiation reduces the participation rate of the low types, i.e., $\theta'(x)$ goes up in all locations. In turn, this induces firm 1 to offer higher qualities to attract some of the higher types who still participate. This narrows the quality gap between firms, leading to more vigorous competition and lower equilibrium prices for all qualities.

In CR's game, firm 2 offers $Q_2 = \left[\bar{\theta}, \infty\right)$ to preempt firm 1 from offering any quality above $\bar{\theta}$. While Bertrand competition drives the prices of any overlapping qualities down to marginal costs, thus making such a deviation unprofitable, this might not necessarily be the case in the presence of horizontal differentiation. Indeed, offering qualities above $\bar{\theta}$ now entails a trade-off for firm 1. On the one hand, there is a direct effect as firm 1 would obtain a positive profit margin out of the high qualities. On the other, there is an strategic effect as firm 2 would react by selling its $\bar{\theta}$ quality at a lower price, which would in turn reduce the profits that firm 1 makes out of its low qualities. For t small enough, the strategic effect always dominates the direct effect, making such a deviation unprofitable.

In sum, introducing small amounts of horizontal differentiation does not contradict CR: firms still prefer to give up profitable opportunities to price discriminate in order to soften competition.

4.2 Search frictions

Instead of adding market frictions through horizontal differentiation, let us now suppose that consumers face search costs. In particular, following Varian (1980), we assume that there is a fraction $\mu < 1$ of consumers who always visit the two firms (the *shoppers*), and hence know where to find the lowest price for each quality. Since the remaining $1 - \mu$ consumers only visit one firm (the *non-shoppers*),[14] they can compare the prices for the qualities sold *within* one firm, but not *across* firms. We assume that the non-shoppers visit one of the two firms with equal probability.[15] In what follows, we use the fraction of non-shoppers $1 - \mu$ as a proxy for search frictions. Accordingly, search frictions are lower the higher μ, with $\mu = 1$ representing a frictionless market.[16]

We can pose a similar question as before: is the equilibrium characterized in Proposition 1 robust to introducing an arbitrarily small amount of search frictions, i.e., $\mu \to 1$? In this case, the answer is no. Intuitively, the presence of non-shoppers increases the incentives to price discriminate: not carrying the full product line stops firms from discriminating not only the shoppers in the gap, but also a wider range of non-shoppers whose preferred qualities are not carried by the firm. In turn, the presence of non-shoppers reduces the incentives to compete: the demands faced by firms become less elastic as price reductions do not attract the non-shoppers. However, this reasoning would seem to suggest that the mass of non-shoppers needs to be large enough for these effects to be strong enough. Yet, and in contrast to the effects of horizontal differentiation, an arbitrarily small amount of search frictions is enough to rule out the equilibrium in Proposition 1.

Proposition 3 *Consider the quadratic-uniform case. If $\mu \to 1$, there does not exist an equilibrium with non-overlapping qualities.*

[14] An implicit assumption is that the fraction μ and the distribution of types are uncorrelated. As we discuss in Section 5.2, our main results do not change if we allow for correlation between both.

[15] In some settings it may be reasonable to assume that non-shoppers observe product lines but not their prices. Accordingly, we have also considered the case in which non-shoppers visit the store that gives them higher expected utility (and split randomly between the two stores in case of symmetry). The main results of the paper are strengthened. See Section 5.2.

[16] Garret *et al.* (2019) introduce search frictions using a more general specification, which encompasses ours. A key property that is common in both specifications is that, with some positive probability, some consumers visit one firm only.

Proof. See the Appendix. ■

Consider the equilibrium in Propositions 1 and 2.[17] In the presence of search frictions, by offering $Q_2 = [\bar{\theta}, \infty)$, firm 2 no longer prevents firm 1 from offering qualities above $\bar{\theta}$. Indeed, firm 1 can extract more rents from some of the non-shopper high types by offering them a quality above $\bar{\theta}$.[18] Firm 1 might be discouraged from doing so if such a deviation intensified competition for the shoppers it serves in equilibrium, as it is the case under horizontal differentiation (for small t). However, as we show in the proof of Proposition 3, it is always possible to find a sufficiently high quality that firm 1 would find it profitable to sell to the non-shoppers without attracting any shoppers. While such a deviation may be enough to rule out the equilibrium in Proposition 1, there is a more profitable deviation for firm 1, which is to offer high qualities, still above $\bar{\theta}$, not only to extract more rents from non-shopper high types, but also to attract some shopper high types. Although this deviation also intensifies the competition for shoppers in the gap, this effect is of second order compared to the increase in the profits made out of the high types. It is as if the gap $q_1^+ < q_2^- = \bar{\theta}$ acted as a buffer. In sum, the non-overlapping equilibrium characterized in Proposition 1 does not survive the introduction of non-shoppers, no matter how few they are.

In sum, under search frictions, there are deviations that allow the low-quality firm to increase its profits from captive consumers, without affecting competition whatsoever; in contrast, under horizontal differentiation, any deviation outside the gap entails, by construction, stealing (high-type) consumers from the rival. In the latter case, the rival would react with deep –as opposed to marginal– price cuts, making such a deviation unprofitable.

5 Equilibrium in Markets with Search Frictions

Since CR's equilibrium prediction does not survive the introduction of search frictions, we now turn to characterizing the new equilibrium. We proceed by backwards induction by first analyzing the second stage (the choice of quality-price menus for given product lines) and then the first stage (the choice of product lines).

[17] To be sure, the deviation that rules out the existence of the equilibrium in Proposition 1 would also rule out CR's equilibrium under their implicit assumption of $\underline{\theta}$ high enough. Indeed, deviation profits would be even higher because under CR's equilibrium firm 1 is even more constrained to extract rents from the non-shoppers.

[18] Note that the equilibrium characterized in Proposition 1 would remain if we were to restrict firm 1 to deviations within the gap, i.e., $q \in (q_1^+, q_2^-]$; for instance, if we adopted the ad-hoc restriction that Q_i has to be an interval.

Consider the choice of quality-price menus. Standard Bertrand arguments imply that there cannot exist an equilibrium in pure strategies: competition for the shoppers would induce firms to slightly undercut prices for all qualities, while rent extraction from the non-shoppers would discourage them from setting prices that are too low. The non-existence of pure strategy equilibria is shared by most models of simultaneous search, starting with Varian (1980).

In Varian's model, firms randomize over a single variable: the price at which they offer an homogeneous product. Since the firm that charges the lowest price attracts all shoppers, it is relatively simple to characterize firms' profits. Matters are more difficult when firms' strategies involve randomizing over quality-price menus. In particular, it might not always be possible to rank the menus offered by the two firms: whereas some consumer types might prefer to buy from firm 1, others might prefer to buy from firm 2 (this occurs when the menus are *non-ordered*). For this reason, and following Garret *et al.* (2019), we restrict attention to equilibria with *ordered* menus, defined as follows:

Definition 1 *(Ordered menus) Consider two menus* $\{q(\theta), p(\theta)\}$ *and* $\{\hat{q}(\theta), \hat{p}(\theta)\}$*, giving utilities* $U(\theta)$ *and* $\hat{U}(\theta)$*, with* $U(\theta') \geq \hat{U}(\theta')$ *for some* $\theta' \in [0, \bar{\theta}]$*. These two menus are ordered if* $U(\theta) \geq \hat{U}(\theta)$ *for all* $\theta \in [0, \bar{\theta}]$*, with strict inequality whenever* $U(\theta) > 0$*. In this case, menu* $\{q(\theta), p(\theta)\}$ *is said to be more generous than menu* $\{\hat{q}(\theta), \hat{p}(\theta)\}$*.*

The menus that firms offer in an ordered-menu equilibrium can be indexed by their generosity, which we denote by x, $\{p_x(\theta), q_x(\theta)\}$. Hence, in equilibrium firms can be thought of as choosing generosity $x \in [\underline{x}, \overline{x}]$ according to a distribution $G(x)$, where $\underline{x}$ and $\overline{x}$ respectively denote the generosity of the *least* and *most* generous menus in the support.

With ordered menus, if firm i chooses a menu of generosity x, it attracts all shoppers (regardless of their valuation) if the rival chooses a less generous menu, an event that occurs with probability $G(x)$. Hence, when a firm chooses a menu with generosity $x \in [\underline{x}, \overline{x}]$, its expected equilibrium profits can be written as

$$\Pi_x = \left(\frac{1-\mu}{2} + G(x)\mu\right)\pi_x, \tag{3}$$

where π_x are the per-consumer expected profits,

$$\begin{aligned} \pi_x &= \int_0^{\bar{\theta}} [p_x(\theta) - C(q_x(\theta))] f(\theta)\, d\theta. \\ &= \int_0^{\bar{\theta}} [\theta q_x(\theta) - U_x(\theta) - C(q_x(\theta))] f(\theta)\, d\theta. \end{aligned}$$

To characterize the mixed strategy equilibrium, we start by assuming that the initial quality range does not constrain firms' offers, i.e., $Q_i = [0, \infty)$, $i = 1, 2$. In a symmetric equilibrium with ordered menus, when a firm offers the least generous menu, the rival firm is offering more generous menus with probability one, i.e., $1-G(\underline{x})=1$. Hence, the firm only serves its fraction $(1-\mu)/2$ of the non-shoppers. Since profits are thus proportional to monopoly profits, firms simply face a monopoly problem when choosing $\underline{x}$. It follows that the optimal least generous menu coincides with the monopoly solution. Since all menus in the support of a mixed strategy equilibrium generate the same expected profits, at any symmetric ordered-menu equilibrium, expected profits are equal to monopoly profits over the non-shoppers.

Lemma 1 *Assume $Q_i=[0,\infty)$, $i=1,2$. For all $\mu\in(0,1)$, at any symmetric equilibrium with ordered menus, the least generous menu is given by the MR solution. Hence, expected equilibrium profits for each firm are $\pi_m(1-\mu)/2$.*

As the share of shoppers goes up, expected equilibrium profits go down from the monopoly solution (when almost all consumers are non-shoppers, $\mu\to 0$) to zero (when almost all consumers are shoppers, $\mu\to 1$).

The lemma above also implies that at the most generous menu that is offered in equilibrium, expected profits must be equal to the monopoly profits from serving the non-shoppers. Since when a firm offers the most generous menu, the rival firm is offering less generous menus with probability one, i.e., $G(\overline{x})=1$, this implies

$$\Pi_{\overline{x}}=\left(\frac{1-\mu}{2}+\mu\right)\pi_{\overline{x}}=\frac{1-\mu}{2}\pi_m,$$

or equivalently,

$$\pi_{\overline{x}}=\frac{1-\mu}{1+\mu}\pi_m.$$

At the most generous menu a firm makes lower per-consumer expected profits than at the least generous menu, but it is more likely to serve more customers. Since per-consumer profits π_x are decreasing in generosity, and the right hand side of the above equation is decreasing in μ, it follows that $\overline{x}$ must be increasing in μ, i.e., the more shoppers there are, the more generous is the most generous menu that is offered in equilibrium.

Interestingly, μ affects the most generous menu, but not the least one (which remains as in the MR solution). Hence, in markets with higher μ (i.e., lower search costs) there is more dispersion in the set of menus offered in equilibrium.

The equilibrium characterization is completed by computing the distribution function that firms use to choose the generosity of their menus.

Lemma 2 *For all $\mu\in(0,1)$, in a symmetric equilibrium with ordered menus, firms choose generosity $x\in[\underline{x},\overline{x}]$ according to*

$$G(x)=\frac{1-\mu}{2\mu}\left(\frac{\pi_m}{\pi_x}-1\right).$$

Proof. It simply follows from equating equation (3) to equilibrium expected profits, equal to the MR's profits of serving the non-shoppers. ■

As μ goes up, more mass is put on more generous menus. In the limit, as $\mu \to 1$, almost all the mass is put at the lower bound, resulting in a menu which is arbitrarily close to the competitive solution.

Last, in order to show that an equilibrium with the above properties indeed exists, we rely on Garret *et al.* (2019).[19]

Proposition 4 *Assume $Q_i = [0, \infty)$, $i = 1, 2$ and consider the quadratic-uniform case. There exists a symmetric equilibrium with ordered menus.*

Proof. See Theorem S1 in the online appendix of Garret *et al.* (2019). ■

At the equilibrium proposed by Garret *et al.* (2019), the highest type always obtains the efficient quality, and its price smoothly goes down under more generous menus. The quality of all the other types is distorted downwards, but quality distortions decrease as firms offer more generous menus. The range of types that are served in equilibrium is enlarged under more generous menus. Indeed, there is a one-to-one mapping between the menu's generosity and the lowest type that is served under the most generous menu.

Garret *et al.* (2019) prove existence, but not uniqueness. They conjecture that their equilibrium is the unique smooth ordered equilibrium of the game, but this does not rule out the possibility that there might exist other equilibria with ordered menus that are not smooth (e.g., with bunching). However, even if this was the case, expected equilibriums profits would remain the same (Lemma 1), which is all that matters for our current purposes; namely, to show existence of an equilibrium with overlapping product lines.

So far, we have restricted attention to unconstrained qualities $Q_i = [0, \infty)$. However, the same analysis would go through for narrower ranges, as long as firms are not constrained from offering the MR qualities. More binding quality ranges would however lead to lower profits as they would constrain firms from extracting monopoly profits out of the non-shoppers.

Lemma 3 *Assume $Q_i = [0, q^+]$ with $q^+ \geq q_c\left(\bar{\theta}\right) = q_m\left(\bar{\theta}\right)$, $i = 1, 2$. For all $\mu \in (0, 1)$, at any symmetric equilibrium with ordered menus, expected equilibrium profits for each firm are $\pi_m(1-\mu)/2$. All other symmetric quality intervals yield strictly lower expected equilibrium profits.*

[19] Garret *et al.* (2019) consider the two-types case. Their online appendix also contains the equilibrium characterization for the continuum type model in the quadratic-uniform case.

Proof. See the Appendix. ■

We are now ready to analyze first stage quality choices. The next proposition characterizes quality choices at the (symmetric) SPNE.

Proposition 5 *For all μ, there exists a SPNE equilibrium with overlapping product lines $Q_i = [0, q^+]$ for $i = 1, 2$, with $q^+ \geq q_c\left(\bar{\theta}\right) = q_m\left(\bar{\theta}\right)$.*

Proof. See the Appendix. ■

Deviations from symmetric product lines are unprofitable whenever they constrain firms from implementing the MR solution. If the deviant carried fewer qualities than at the MR solution, the deviant would not be able to obtain monopoly profits over the non-shoppers, making such a deviation unprofitable. Furthermore, there cannot exist symmetric equilibria with narrower quality ranges as firms would find it optimal to enlarge their first stage product lines until they no longer constrain their second stage choices. The intuition is simple: a firm that deviated would be able to at least obtain monopoly profits over the non-shoppers, regardless of the menus offered by the rival firm.

To conclude, in the presence of search frictions, no matter how big or small, the symmetric SPNE involves overlapping quality choices that do not constrain firms from implementing the monopoly solution over the non-shoppers. The mass of shoppers determines how close the equilibrium is to the monopoly solution ($\mu = 0$) or to the competitive solution ($\mu \to 1$). In all cases, the equilibrium involves overlapping quality choices over the full range.

5.1 The non-overlapping versus the overlapping equilibria

Search frictions affect outcomes through two channels: they impact price and quality when the overlapping equilibrium prevails (as already discussed in Garret *et al.* (2019)), and they impact product lines when the equilibrium switches from the non-overlapping to the overlapping type when arbitrarily small search frictions are introduced (Proposition 5).

Due to the latter effect, the pattern of expected prices for given qualities depicts a discontinuity when search frictions are arbitrarily small. Indeed, at the non-overlapping equilibrium firms are able to sustain prices that are strictly above marginal costs for all qualities on sale. In contrast, at the overlapping equilibrium with arbitrarily small search frictions, all qualities are offered at marginal cost with probability close to one. Using the terminology of Armstrong (2015), non-shoppers create a positive search externality to the shoppers as these end up paying lower prices while the range of qualities on offer is enlarged. However, as search frictions become more important, higher prices are played with greater probability, eventually leading to prices that are higher than under the non-overlapping equilibrium with no search costs. Hence, the conventional wisdom that search frictions lead to higher prices applies in this model, but only when search frictions do not change equilibrium product lines, i.e., everywhere except in the limit when search frictions become arbitrarily small.

The range of qualities that are actually bought under the non-overlapping equilibrium is much narrower than at the overlapping equilibrium because the preferred qualities of those consumers in the "gap" are not available. Hence, introducing an arbitrarily small amount of search costs also implies a discontinuous jump in the range of qualities bought.

Last, putting the price and quality impacts together, consumers are better off with mild search frictions than in frictionless markets: prices are lower and there is more product variety. However, when search frictions are sufficiently high, consumers are faced with a trade-off as prices are higher than in the absence of search frictions but there is more product variety. Accordingly, there exists a level of search frictions above which consumers are worse off than in frictionless markets, and vice-versa.

5.2 Extensions and variations

In this section we have characterized quality and price choices in markets with search costs under three assumptions which we now seek to relax: (i) duopoly; (ii) search cannot be conditioned on product line choices (as these were assumed non-observable prior to search); and (iii) consumers' search frictions and quality preferences are uncorrelated. Our discussion focuses on the existence of the "overlapping" equilibrium.

N **symmetric firms oligopoly** A similar logic as in the duopoly case also allows to conclude that all firms carrying all qualities constitutes a SPNE for all $\mu < 1$. In the second stage, the least generous menu in the mixed strategy equilibrium is given by the monopoly solution given that at this menu the firm only sells to the non-shoppers (since firms are symmetric, there cannot be a mass at the least generous menu). Thus, equilibrium profits are a fraction $(1 - \mu)/N$ of monopoly profits just as in the duopoly case. Alternatively, if one firm deviates by dropping one or more qualities, the deviant (weakly) reduces its profits as it will not be able to implement the monopoly solution. Hence, the presence of shoppers restores the monopolist's incentives to carry all qualities just as in the duopoly case.

Observable product choices and directed search by the non-shoppers In the main model we assumed that consumers do not observe product lines prior to visiting the firms. In particular, we assumed that the non-shoppers visit one of the two firms with equal probability, regardless of their product choices. Instead, suppose now that non-shoppers visit the store that gives them higher expected utility, given firms' (observable) product choices and expected prices.[20] Allowing search to be conditioned on product choices would strengthen our main result: when directed search is allowed, offering more product variety would allow firms to not only better discriminate, but also to attract more non-shoppers.

Directed search by the non-shoppers only affects pricing when firms have chosen asymmetric product lines (with symmetric product lines, expected prices are also symmetric so it is irrelevant whether search is directed or random). We have formally studied this problem elsewhere (Fabra and Montero, 2017), but in a simpler setting of two types of consumers and two qualities (high and low). Suppose that one firm carries the low quality version and the other carries both versions. The first observation is that directed search requires expected prices for the low quality to be equal across firms, so in equilibrium non-shopper low types are indifferent as to which store to visit. Second, all non-shopper high types visit the multi-product firm not only because expected prices for the low quality version are equal across firms but also because incentive compatibility makes them indifferent between the two qualities.

The directed search outcome just described differs from the one when non-shoppers split evenly between the two firms (i.e., when product lines are not observable). In this latter case, the multi-product firm charges lower prices for the low quality good. This more aggressive pricing is explained by the multi-quality firm's ability to segment non-shoppers. Now, to rebalance firms' pricing incentives from this latter case to the case of directed search, more than half of the non-shopper low types must visit the multi-product store until their expected prices converge, ultimately reducing the single-product firm's market share and profit. This implies that the incentives to deviate from the case in which both firms carry both qualities to carry just the low quality one are necessarily weaker under directed search. A similar reasoning applies to a deviation to carry just the high quality version.

We see no reason to expect that this reasoning would fail in our more general setting of many qualities and consumer types, but it would be complex to prove existence of equilibrium. Thus, we believe that our main conclusion –namely, that the "overlapping" equilibrium is robust for all $\mu < 1$– remains valid regardless of whether product lines are observable (and there is directed search by the non-shoppers) or not.

[20] This interpretation of non-shoppers as sophisticated buyers is closer to that in the clearing-house model *à la* Baye and Morgan (2001).

Correlation between search frictions and quality preferences Last, we have so far assumed that shoppers and non-shoppers are equally likely to be either high or low types. However, this may not hold in practice. For instance, if low types are lower income consumers with more time to search, then the non-shoppers are more likely to be high types.[21] Alternatively, if high types enjoy shopping for their preferred (high quality) product, then non-shoppers are more likely to be low types. Ultimately, this is an empirical question whose answer may vary depending on the type of product or context considered. However, as far as the predictions of the model are concerned, it is inconsequential whether the correlation between search frictions and quality preferences is positive, negative or non-existent.[22]

To formalize this, one can assume that the fraction of shoppers might vary across consumer types, i.e., $\mu(\theta)$ represents the fraction shoppers of type θ, with $\mu = \int \mu(\theta) f(\theta) d\theta$. If $\mu(\theta)$ decreases in θ, there is positive correlation between search frictions and quality types as the high types are less likely to be shoppers (i.e., higher types have higher search costs). The analysis of product and price choices without search frictions remains intact since all consumers are shoppers by definition. As for the analysis with search frictions, expected profits at a symmetric equilibrium with ordered menus remain proportional to monopoly profits, thus implying that the incentive structure remains unchanged. As such, the "overlapping" equilibrium always exists just as in the case with no correlation between search and quality preferences.

6 Conclusions

We have analyzed a duopoly model of quality choice followed by imperfect price competition. We have found that the source of market power affects the relationship between competition and price discrimination. In markets with a small amount of horizontal differentiation, it is still true that firms are able to soften competition by carrying non-overlapping product lines, as in the seminal paper of Champsaur and Rochet (1989). In contrast, an arbitrarily small amount of search frictions is enough to overturn this predic-

[21] This negative correlation would be consistent with the evidence in Kaplan and Menzio (2016), as employed workers have less time to search for low prices than unemployed workers.

[22] One may also argue that being a shopper should be an equilibrium decision; for example, if consumers, after learning about their types, had the option to pay some "information-acquisition" cost to become shoppers. We believe that our main results would prevail if this information-acquisition cost were distributed over the population of consumers such that, regardless of their type, some fraction of consumers decided to remain uninformed in equilibrium. If, for some reason, all high types decided to become informed in equilibrium, the schedules in the overlapping equilibrium may be distorted downwards, i.e., the least generous schedule may no longer be the monopoly schedule but something slightly more competitive (i.e., more generous).

tion. Instead, there always exist an equilibrium with overlapping product lines, that gives rise to otherwise stronger competition than under non-overlapping quality choices. For this reason, analyzing the price effects of search frictions without endogenizing product lines can sometimes lead to overestimating their anticompetitive effects.

Admittedly, there are several motives other than the ones studied in this paper that shape firms' product choices. In particular, throughout the analysis we have assumed that firms do not incur any fixed cost of carrying a product. This modelling choice was meant to highlight the strategic motives underlying product choice. However, fixed costs of carrying a product (which could arguably be higher for high quality products),[23] could induce firms to offer fewer and possibly non-overlapping products. Our prediction is not that in markets with search costs and homogeneous products competitors should always carry overlapping product lines. Rather, our analysis suggests that if their product lines do not overlap, it must be for reasons other than firms' attempts to soften competition through product choice; for instance, due to the presence of fixed costs.

To the extent that firms could collude to coordinate their product choices (as reported by Sullivan (2017) in the context of the super-premium ice cream market),[24] competition authorities should remain vigilant if competitors' product lines do not overlap - particularly so in markets in which fixed costs (at the product level) are not relevant, there is substantial product differentiation or consumers find it costly to search.

Appendix: Proofs

Proof of Proposition 1 Suppose that in the first stage firms choose $Q_1 = [0, q_1^+]$ and $Q_2 = [q_2^-, \bar{\theta}]$, with $q_1^+ \leq q_2^-$, respectively. As long as q_1^+ and q_2^- are not too far apart (in a way to be made precise shortly), equilibrium prices $p_1^+ \equiv p_1(q_1^+)$ and $p_2^- \equiv p_2(q_2^-)$ at the second stage can be obtained from solving the auxiliary pricing game where firms carry

[23] In some cases, such costs can be substantial, e.g. firms have to advertise that they are carrying an additional product, or the transaction costs of dealing with an additional provider can sometimes be high. The marketing literature has analyzed several factors explaining the limited number of products sold per firm. For instance, Villas-Boas (2004) analyzes product line decisions when firms face costs of communicating about the different products they carry to their customers. They show that costly advertising can induce firms to carry fewer products as well as to charge lower prices for their high-quality goods.

[24] See also the *NY Times* note quoted in the paper. Although it is difficult to divide "smooth" and "chunky" flavors in low and high-quality options, the logic of our result may apply as well. The ice-cream company that focuses on chunky flavors may need to reprice its existing offer downwards if it decides to also carry smooth flavors and compete head-to-head on these flavors with the rival company just carrying them. But this is profitable as long as exist some fraction of smooth non-shoppers.

a single quality (see CR's Proposition 1),

$$p_1^+ = \arg\max_{p_1}(p_1 - C(q_1^+))\left(F(\tilde{\theta}) - F(\theta')\right) \tag{4}$$

and

$$p_2^- = \arg\max_{p_2}(p_2 - C(q_2^-))\left(1 - F(\tilde{\theta})\right), \tag{5}$$

where $\tilde{\theta} = (p_2 - p_1)/(q_2^- - q_1^+)$ is the shopper indifferent between options (q_1^+, p_1^+) and (q_2^-, p_2^-) and $\theta' = p_1/q_1^+$ is the last shopper buying from firm 1. Solving the system of first-order conditions yields

$$p_1^+ = C(q_1^+) + (F(\tilde{\theta}) - F(\theta'))\frac{q_1^+(q_2^- - q_1^+)}{f(\tilde{\theta})(q_2^- - q_1^+) + f(\theta')q_1^+} \tag{6}$$

and

$$p_2^- = C(q_2^-) + (1 - F(\tilde{\theta}))\frac{(q_2^- - q_1^+)}{f(\tilde{\theta})}. \tag{7}$$

In addition to these two prices, we need equilibrium schedules for qualities other than $q_1^+$and q_2^-. Firm 2's schedule corresponds to the Mussa-Rosen schedule reviewed in Section 3, the only difference being that $U(\theta_2^*)$, the utility of the lowest type ($\theta_2^* \geq \tilde{\theta}$) served under the schedule, is no longer zero but equal to $U(\theta_2^*) = \theta_2^* q_2^- - p_2^-$ (firm 1's presence has increased the low end outside option of consumers buying from firm 2). So, using (1), firm 2's price schedule can be written as

$$p_2(\theta) = \theta q_2(\theta) - U(\theta) = \theta q_2(\theta) - \int_{\theta_2^*}^{\theta} q_2(s)\,ds - U(\theta_2^*),$$

which is then used to obtain firm 2's optimal quality schedule,

$$C'(q_2(\theta)) = \theta - \frac{F(\bar{\theta}) - F(\theta)}{f(\theta)}.$$

For the quadratic-uniform setting, this schedule reduces to $q_2(\theta) = q_m(\theta) = 2\theta - \bar{\theta}$ for all $\theta \in [\theta_2^*, \bar{\theta}]$ and zero otherwise.

We still need a expression for θ_2^* as a function of first-stage variables. Using the "smooth-pasting" condition $q_2(\theta_2^*) = q_2^-$, we obtain $\theta_2^* = (q_2^- + \bar{\theta})/2$. From here, and using (6) and (7), we can express firm 2's (first-stage) payoff as a function of q_1^+ and q_2^- as follows

$$\Pi_2(q_1^+, q_2^-) = \int_{\theta_2^*}^{\bar{\theta}} \left[\left(\theta - \frac{F(\bar{\theta}) - F(\theta)}{f(\theta)}\right) q_2(\theta) - C(q_2(\theta))\right] f(\theta)\,d\theta$$
$$- U(\theta_2^*)\left(F(\bar{\theta}) - F(\theta_2^*)\right) + \left(p_2^- - C(q_2^-)\right)\left[F(\theta_2^*) - F(\tilde{\theta})\right].$$

Firm 1's price schedule $p_1(\theta)$ is more involved because there are two outside options to be handled. While low types still have to be prevented from buying the low quality (outside) option (for a payoff that has been normalized to zero), high types now have to be prevented from buying a high quality option offered by firm 2, in particular q_2^-. Formally, there is a type $\hat{\theta} \leq \theta_1^{**} \leq \tilde{\theta}$, where θ_1^{**} is the highest-type consumer buying from firm 1 under the price schedule $p_1(\theta)$, such that, by incentive compatibility, the utility of types $\theta < \hat{\theta}_1$ is still given by (1), while the utility of types $\theta > \hat{\theta}_1$ now takes the form (preventing lower types from mimicking higher types)

$$U(\theta) = U(\theta_1^{**}) - \int_\theta^{\theta_1^{**}} q_1(s)\, ds$$

for $\theta \in [\hat{\theta}_1, \theta_1^{**}]$. Combining this latter with (1) for $\theta \in [\theta_1^*, \hat{\theta}_1]$, where θ_1^* is the lowest type being served, we proceed as before to obtain firm 1's optimal quality schedule

$$C'(q_1(\theta)) = \theta - \frac{F(\hat{\theta}_1) - F(\theta)}{f(\theta)}.$$

For the quadratic-uniform setting, this reduces to $q_1(\theta) = 2\theta - \hat{\theta}_1$ for all $\theta \in [\theta_1^*, \theta_1^{**}]$. Only type $\hat{\theta}_1$ obtains the efficient quality. Types $\theta > \hat{\theta}_1$ are offered inefficiently high qualities, while types $\theta < \hat{\theta}_1$ are offered inefficiently low qualities.

In addition to the smooth-pasting condition $q_1(\theta_1^{**}) = 2\theta_1^{**} - \hat{\theta}_1 = q_1^+$ and $U(\theta_1^{**}) = \theta_1^{**} q_1^+ - p_1^+$, we also know from $U(\theta_1^*) = 0$ that $q_1(\theta_1^*) = 2\theta_1^* - \hat{\theta}_1 = 0$ and $U(\theta_1^{**}) = \int_{\theta_1^*}^{\theta_1^{**}} q_1(\theta) f(\theta) d\theta$. From here, we can express firm 1's (first-stage) payoff as a function of q_1^+ and q_2^- as follows

$$\Pi_1(q_1^+, q_2^-) = \int_{\theta_1^*}^{\theta_1^{**}} \left[\left(\theta - \frac{F(\hat{\theta}_1) - F(\theta)}{f(\theta)} \right) q_1(\theta) - C(q_1(\theta)) \right] f(\theta)\, d\theta$$
$$- U(\theta_1^{**}) \left(F(\theta_1^{**}) - F(\hat{\theta}_1) \right) + [p_1^+ - C(q_1^+)] \left(F(\tilde{\theta}) - F(\theta_1^{**}) \right).$$

Moving backwards to the first stage to solve the system $q_1^+ = \arg\max_{q_1} \Pi_1(q_1, q_2^-)$ and $q_2^- = \arg\max_{q_2} \Pi_2(q_1^+, q_2)$, we quickly arrive at the corner $q_1^+ = q_1(\theta_1^{**}) = \theta_1^{**} = \hat{\theta}_1 = (2 - \sqrt{2})\bar{\theta}$ and $q_2^- = \bar{\theta}$, where $\partial \Pi_1(q_1^+, q_2^-)/\partial q_1 < 0$ and $\partial \Pi_2(q_1^+, q_2^-)/\partial q_2 > 0$. The reason q_1^+ cannot be reduced any further despite $\partial \Pi_1(q_1^+, q_2^-)/\partial q_1 < 0$ is because when $q_1^+ < (2 - \sqrt{2})\bar{\theta}$ (while holding q_2^- fixed at $\bar{\theta}$), p_1^+ is no longer governed by the system (4) and (5), but by a MR schedule with no distortion at the top of its quality range, i.e., $q_1(q_1^+) = q_m(q_1^+) = q_1^+$ and $p_1(q_1^+) = q_1^+ q_1(q_1^+) - \int_{\theta_1^*}^{q_1^+} q_1(s)\, ds = 3(q_1^+)^2/4$ (recall that from $U(\theta_1^*) = 0$, we have $\theta_1^* = q_1^+/2$). We say that at this point q_1^+ is too far apart from q_2^-, so that the schedule $q_1(\theta)$ offered in equilibrium by firm 1 to consumers $\theta \leq q_1^+$ is not affected at all by firm 2's offer (and CR's Proposition 1 no longer applies).[25]

When firm 1's schedule is governed by a MR schedule, firm 1's payoff $\Pi_1(q_1^+, q_2^-)$ needs to be modified accordingly: $\theta_1^{**} = \hat{\theta}_1 = q_1^+$, $\theta_1^* = q_1^+/2$, $q_1(\theta) = 2\theta - q_1^+$ and $p_1^+ = 3(q_1^+)^2/4$. Firm 2's payoff also needs to be modified slightly in that p_2^- is no longer given by (7) but needs to be obtained directly from (5). Solving the system $q_1^+ = \arg\max_{q_1} \Pi_1(q_1, q_2^-)$ and $q_2^- = \arg\max_{q_2} \Pi_2(q_1^+, q_2)$ for these new payoff functions, we quickly arrive at the same corner: $q_1^+ = (2-\sqrt{2})\bar{\theta}$ and $q_2^- = \bar{\theta}$, where $\partial\Pi_1(q_1^+, q_2^-)/\partial q_1 > 0$ and $\partial\Pi_2(q_1^+, q_2^-)/\partial q_2 > 0$. The reason q_1^+ cannot be increased any further despite $\partial\Pi_1(q_1^+, q_2^-)/\partial q_1 > 0$ is because when $q_1^+ > (2-\sqrt{2})\bar{\theta}$ (while holding q_2^- fixed at $\bar{\theta}$), p_1^+ is again governed by the system (4) and (5).[26]

The proof concludes calling CR's Proposition 5: if the pair of quality ranges $Q_1 = [0, q_1^+ = (2-\sqrt{2})\bar{\theta}]$ and $Q_2 = [q_2^- = \bar{\theta}, \bar{\theta}]$ constitutes a SPNE of the quality game, then the pair of quality ranges $Q_1 = [0, q_1^+ = (2-\sqrt{2})\bar{\theta}]$ and $Q_2' = [q_2^- = \bar{\theta}, +\infty)$ also constitutes a SPNE of the quality game, leading to the exact same pricing schedules and payoffs.

Proof of Proposition 2 There are two relevant deviations from the equilibrium qualities $Q_1^* = \left[0, q_1^+\right]$ and $Q_2^* = \left[q_2^-, \infty\right)$, with $q_1^+ = (2-\sqrt{2})\bar{\theta}$ and $q_2^- = \bar{\theta}$, to be considered. The first is a deviation "within the quality gap", where firms deviate to either $q_1^+ \neq (2-\sqrt{2})\bar{\theta} \leq q_2^-$, $q_2^- \neq \bar{\theta} \geq q_1^+$ or both. The second is a deviation "outside the gap", where firm 1 decides not only to carry $Q_1 = \left[0, q_1^+\right]$ but also additional qualities above $\bar{\theta}$. There is no need to look at deviations by firm 2 outside the gap. The reason is simple: firm 1 is carrying the whole range of low qualities, so any deviation by firm 2 to carry $q < q_1^+$ would drive the price for such quality close to cost, as $t \to 0$.

To analyze the first type of deviation, suppose that in the first stage firms choose $Q_1 = [0, q_1^+]$ and $Q_2 = [q_2^-, \bar{\theta}]$, with $q_1^+ \leq q_2^-$, respectively. As long as q_1^+ and q_2^- are not too far apart (in a way to be made precise shortly), equilibrium prices $p_1^+ \equiv p_1(q_1^+)$ and $p_2^- \equiv p_2(q_2^-)$ at the second stage can still be obtained from solving the auxiliary pricing game where firms carry a single quality (see CR's Proposition 1),

$$p_1^+ = \arg\max_{p_1}(p_1 - C(q_1^+)) \int_0^1 \left(F(\tilde{\theta}(x)) - F(\theta'(x))\right) dx \tag{8}$$

[25] Note that when q_1^- is sufficiently low, i.e., when $q_1^- < 0.53\bar{\theta}$ in our quadratic-uniform setting (while holding q_2^- fixed at $\bar{\theta}$), prices are again governed by the system (4) and (5) because from now on firm 1 finds it optimal to offer a single quality, q_1^+, and give up on any segmentation upon lower types. Segmenting lower types, which are not that valuable anymore, would only introduce (lower) qualities that would force firm 1 to leave more rents with higher types.

[26] Note that at the equilibrium, firms' payoffs can still be decomposed as in CR's Proposition 4 because their Proposition 1 pricing equilibrium is just valid.

and

$$p_2^- = \arg\max_{p_2}(p_2 - C(q_2^-)) \int_0^1 \left(1 - F(\tilde{\theta}(x))\right) dx, \tag{9}$$

where

$$\tilde{\theta}(x) = \frac{p_2^- - p_1^+ + t(1-2x)}{q_2^- - q_1^+} \tag{10}$$

is the consumer located at x who is indifferent between options (q_1^+, p_1^+) and (q_2^-, p_2^-). Clearly, $\tilde{\theta}(x)$ is decreasing in x, i.e., the further away from firm 1's location, the closer to firm 1 the indifferent consumer is. Likewise,

$$\theta'(x) = \frac{p_1^+ + tx}{q_1^+}$$

is the last consumer buying from firm 1. Now, $\theta'(x)$ is increasing in x.

Solving the system of first-order conditions yields

$$p_1^+ = C(q_1^+) + q_1^+(q_2^- - q_1^+) \frac{\int_0^1 (F(\tilde{\theta}(x)) - F(\theta'(x)))dx}{\int_0^1 (f(\tilde{\theta}(x))(q_2^- - q_1^+) + f(\theta'(x))q_1^+)dx} \tag{11}$$

and

$$p_2^- = C(q_2^-) + (q_2^- - q_1^+) \frac{\int_0^1 (1 - F(\tilde{\theta}(x)))dx}{\int_0^1 f(\tilde{\theta}(x))dx}. \tag{12}$$

Since for the uniform distribution we have that

$$\int_0^1 (F(\tilde{\theta}(x)) - F(\theta'(x)))dx = \left(\frac{p_2^- - p_1^+}{q_2^- - q_1^+} - \frac{p_1^+ + t/2}{q_1^+} \right) \frac{1}{\bar{\theta}}$$

and

$$\int_0^1 (1 - F(\tilde{\theta}(x)))dx = \left(\bar{\theta} - \frac{p_2^- - p_1^+}{q_2^- - q_1^+} \right) \frac{1}{\bar{\theta}},$$

expressions (11) and (12) simplify to

$$p_1^+ = \frac{1}{2} \left[C(q_1^+) - \frac{q_2^- - q_1^+}{2q_2^-} t + \frac{q_1^+}{q_2^-} p_2^- \right] \tag{13}$$

and

$$p_2^- = \frac{1}{2} \left[C(q_2^-) + \bar{\theta} q_2^- - \bar{\theta} q_1^+ + p_1^+ \right]. \tag{14}$$

It is clear that prices are decreasing in t for given q_1^+ and q_2^-. The reason is that the introduction of "transportation costs" has reduced firm 1's inframarginal consumers from below, those with low valuations who no longer consume. Given this, it pays firm 1 to be more aggressive, and firm 2 to respond accordingly.

In addition to these two prices, we need equilibrium schedules for qualities other than q_1^+ and q_2^-. As before, firm 2's schedule corresponds to a Mussa-Rosen (MR) schedule.

Note from Proposition 1 that $\theta_2^* > \tilde{\theta}$, so the fact that $t \to 0$ implies that θ_2^* will continue to be invariant to x and that $\theta_2^* > \tilde{\theta}(x)$ for all x. So, proceeding as in Proposition 1, firm 2's (first-stage) payoff as a function of q_1^+ and q_2^- can be expressed as

$$\Pi_2(q_1^+, q_2^-) = \int_{\theta_2^*}^{\bar{\theta}} \left[\left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) q_2(\theta) - C(q_2(\theta)) \right] f(\theta)\, d\theta$$
$$- U(\theta_2^*)(1 - F(\theta_2^*)) + \left(p_2^- - C(q_2^-) \right) \int_0^1 (F(\theta_2^*) - F(\tilde{\theta}(x)) dx.$$

Because of the uniform distribution, this payoff function turns out to be exactly the same as the one in the proof of Proposition 1, i.e., $\int_0^1 F(\tilde{\theta}(x))dx = (p_2^- - p_1^+)/\bar{\theta}(q_2^- - q_1^+)$.

Firm 1's price schedule $p_1(\theta)$ is more involved, not only because of the two outside options we discussed before, but also because the last type being served now depends on its location x (see expression (10)). Suppose that firm 1 offers the schedule $\{q_1(\theta), p_1(\theta)\}$. Let us denote by θ_1^* the last consumer who buys from firm 1 regardless of her location, that is,

$$\theta_1^* q_1(\theta_1^*) - p_1(\theta_1^*) - tx \geq 0$$

for all x. Since the term tx only affects the decision to consume or not, but not what quality to consume, firm 1's problem is essentially the same as before for all $\theta \in [\theta_1^*, \theta_1^{**}]$: to find an IC schedule $\{q_1(\theta), p_1(\theta)\}$ where $U(\theta_1^*) = \theta_1^* q_1(\theta_1^*) - p_1(\theta_1^*) = t$. We know that all consumers with $\theta \geq \theta_1^*$ will consume according to $\{q_1(\theta), p_1(\theta)\}_{\theta \in [\theta_1^*, \theta_1^{**}]}$ regardless of their location x. Note that since $t \to 0$, there will be a small fraction of consumers $\theta < \theta_1^*$ still being served, those located close to firm 1 (small x), according to a declining pdf.

Before proceeding, we need an additional condition that allows us to pin down $q_1(\theta_1^*)$. Recall that in Proposition 1 it was not only optimal to set $U(\theta_1^*) = 0$ but also to set $q_1(\theta_1^*) = 0$. The reason for the latter is that setting $q_1(\theta_1^*) = 0$ reduces the information rents that need to be given to all $\theta \geq \theta_1^*$. As $t \to 0$, these information rents are more important than the profits that can be made on types below θ_1^*. Therefore, the equivalent condition here, which is to set $U(\theta_1^*) = t$ while keeping information rents for all $\theta \geq \theta_1^*$ at a minimum, is to reduce $q_1(\theta_1^*)$ as much as possible, which means selling it at cost, that is

$$\theta_1^* q_1(\theta_1^*) - \frac{1}{2}(q_1(\theta_1^*))^2 = t. \tag{15}$$

Equipped with this boundary condition and proceeding similarly as in the proof of

Proposition 1, we arrive at

$$\Pi_1(q_1^+, q_2^-) = \int_{\theta_1^*}^{\theta_1^{**}} \left[\left(\theta - \frac{F(\hat{\theta}_1) - F(\theta)}{f(\theta)} \right) q_1(\theta) - C(q_1(\theta)) \right] f(\theta)\, d\theta$$
$$- U(\theta_1^*) \left(F(\hat{\theta}_1) - F(\theta_1^*) \right) - U(\theta_1^{**}) \left(F(\theta_1^{**}) - F(\hat{\theta}_1) \right)$$
$$+ \left[p_1^+ - C(q_1^+) \right] \int_0^1 \left(F(\tilde{\theta}(x)) - F(\theta_1^{**}) \right) dx.$$

where $U(\theta_1^{**}) = U(\theta_1^*) + \int_{\theta_1^*}^{\theta_1^{**}} q_1(\theta) f(\theta) d\theta$ and $U(\theta_1^*) = t$.

We now move backwards to the first stage to solve the system $q_1^+ = \arg\max_{q_1} \Pi_1(q_1, q_2^-)$ and $q_2^- = \arg\max_{q_2} \Pi_2(q_1^+, q_2)$. Using (13), (14) and (15), and the fact that $t \to 0$, we quickly arrive at the corner $q_2^- = \bar{\theta}$ and $q_1^+ = q_1(\theta_1^{**}) = \theta_1^{**} = \hat{\theta}_1 = \mathring{q}$, where $\mathring{q}$ solves

$$\frac{3}{4}\mathring{q}^2 - t + \frac{t^2}{\mathring{q}^2} = \frac{1}{8\bar{\theta} - 2\mathring{q}} \left(3\mathring{q}\bar{\theta}^2 - 2t\bar{\theta} + 2\mathring{q}t \right) \tag{16}$$

(note that $\mathring{q} = (2-\sqrt{2})\bar{\theta}$ when $t = 0$, as in Proposition 1). At the corner $q_1^+ = \mathring{q}$ and $q_2^- = 1$, we have that $\partial \Pi_1(q_1^+, q_2^-)/\partial q_1^+ < 0$ and $\partial \Pi_2(q_1^+, q_2^-)/\partial q_2^- > 0$. The reason q_1^+ cannot be reduced any further despite $\partial \Pi_1(q_1^+, q_2^-)/\partial q_1 < 0$ is that when $q_1^+ < \mathring{q}$ (while holding q_2^- fixed at 1), p_1^+ is no longer governed by the system of equations (8) and (9), but by a MR schedule with no distortion at the top of its quality range. In this sense, q_1^+ is now too far apart from q_2^-, meaning that the schedule $q_1(\theta)$ offered in equilibrium by firm 1 to consumers $\theta \leq q_1^+$ is not affected at all by firm 2's offer (and CR's Proposition 1 no longer applies).

When firm 1's schedule is governed by a MR schedule, firm 1's payoff $\Pi_1(q_1^+, q_2^-)$ needs to be modified accordingly: $\theta_1^{**} = \hat{\theta}_1 = q_1^+$, $q_1(\theta) = 2\theta - q_1^+$, $\theta_1^* = q_1^+/2 + t/q_1^+$, and $p_1^+ = 3(q_1^+)^2/4 - t + t^2/(q_1^+)^2$. Firm 2's payoff also needs to be modified slightly in that p_2^- is no longer given by (12) but needs to be obtained directly from (9). Solving the system $q_1^+ = \arg\max_{q_1} \Pi_1(q_1, q_2^-)$ and $q_2^- = \arg\max_{q_2} \Pi_2(q_1^+, q_2)$ for these new payoff functions, we quickly arrive at the same corner: $q_1^+ = \mathring{q}$ and $q_2^- = \bar{\theta}$, where $\partial \Pi_1(q_1^+, q_2^-)/\partial q_1 > 0$ and $\partial \Pi_2(q_1^+, q_2^-)/\partial q_2 > 0$. The reason why q_1^+ cannot be increased any further despite $\partial \Pi_1(q_1^+, q_2^-)/\partial q_1 > 0$ is because when $q_1^+ > \mathring{q}$ (while holding q_2^- fixed at $\bar{\theta}$), p_1^+ is again governed by the system (8) and (9).

So far there are no fundamental changes relative to Proposition 1: as $t \to 0$ the equilibrium characterized above converges to the one in Proposition 1. Important to notice, however, is that the introduction of $t > 0$ increases the equilibrium value of $q_1^+ = \mathring{q}$. An increase in horizontal differentiation closes the quality gap —firm 1 becomes more aggressive because of fewer inframarginal consumers from lower participation of lower types—, which, in turn, prompts lower equilibrium prices for all qualities.[27,28]

We now move on to analyze deviations "outside the gap", that is, whether firm 1, the one carrying the lower qualities, has incentives to add to her lineup qualities $q > \bar{\theta}$. To explore this possibility we can simply focus on single-quality deviations, for the case when $Q_2 = \{\bar{\theta} = 1\}$ as opposed to when $Q_2 = [\bar{\theta}, \infty)$. The reason is that if firm 1 cannot profitably deviate when firm 2 only carries one quality, she cannot profitably deviate when firm 2 carries more than one. In other words, allowing for the full range of qualities would make the deviation of firm 1 less profitable as the firm would encounter more competition.

Our strategy of proof is simple: we will show that firm 1 is strictly worse off (i.e., it suffers from a discrete, as opposed to marginal, drop in profits) by carrying any $q > \bar{\theta}$ when $t = 0$, which is enough to show that firm 1 would not want to deviate either when $t \to 0$. This proof also serves to show that $Q_1 = [0, q_1^+]$ and $Q_2 = \{\bar{\theta}\}$ is an equilibrium of the CR game, i.e., when $t = 0$, leading to the same prices and payoffs as in Proposition 1. In other words, it is not necessary for firm 2 to carry qualities that it will not be selling in equilibrium.

Accordingly, suppose that firm 1 deviates at stage 1 to also carry $q_1^{++} > \bar{\theta}$ while keeping q_1^+ as in Proposition 1 (shortly it will become clear that our conclusion does not change if firm 1 is also allowed to adjust q_1^+ at the time of the deviation). We start by finding the highest value that q_1^{++} can take, that is, the value at which firm 1 is just able to attract the highest type by selling q_1^{++} at cost. Denote that value by $\hat{q}$:

$$\bar{\theta}\hat{q} - C(\hat{q}) = \bar{\theta}q_2(\bar{\theta}) - p_2(\bar{\theta}) \tag{17}$$

where, from Proposition 1, $q_2(\bar{\theta}) = \bar{\theta}$ and $p_2(\bar{\theta}) = (2 - \sqrt{2})\bar{\theta}$. Solving (17) we obtain $\hat{q} = \sqrt{2}\bar{\theta}$. Thus, any relevant (i.e., potentially profitable) deviation by firm 1 must entail $q_1^{++} \in (\bar{\theta}, \sqrt{2}\bar{\theta})$.

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[27] For example, when $t = 0.01$, we have that $p_2(q_2 = \bar{\theta}) = 0.580\bar{\theta}^2 < 0.586\bar{\theta}^2$ and $p_1(q_1 = (2 - \sqrt{2})\bar{\theta}) = 0.254\bar{\theta}^2 < 0.257\bar{\theta}^2$.

[28] Let us stress that the analysis presented here is only valid for small values of t, which is the focus of Proposition 2. Note that for higher values of t we would need to solve the schedule

$$C'\left(q_1\left(\theta\right)\right) = \theta - \frac{F(\theta_1^*) - F\left(\theta\right)}{f\left(\theta\right)}.$$

for all $\theta < \theta_1^*$, where $F(\theta)$ and $f(\theta)$ are not longer the cdf and pdf of a uniform distribution, but rather depend on the location of the last consumer θ being served, say $x'(\theta)$, so that $f(\theta) = x'(\theta)/\bar{\theta}$. The difficulty is that $x'(\theta)$ endogenously depends on the schedule $\{q_1(\theta), p_1(\theta)\}$

$$x'(\theta) = (\theta q_1(\theta) - p_1(\theta))/t$$

making the solution to firm 1's problem a non trivial fixed-point problem.

Suppose then that firm 1 deviates to also carry one quality q_1^{++} in the range $(\bar{\theta}, \sqrt{2}\bar{\theta})$ and denote by p_1^{++} the price that firm 1 will charge for this quality in the pricing stage. As we move to this stage, notice first that price p_1^{+} will no longer be governed by a MR schedule since $p_2(\bar{\theta}) \equiv p_2$ will now be strictly lower. Neither can we rely on equations (6) and (7) to find p_1^{+} and p_2 since firm 2 is now also facing competition from both below and above. Therefore, equilibrium prices are to be obtained directly, from the first-order conditions.

We know that

$$\tilde{\theta} = \frac{p_2 - p_1^{+}}{\bar{\theta} - q_1^{+}} \tag{18}$$

is the indifferent consumer within the gap (i.e., between purchasing $q_1^{+} = (2-\sqrt{2})\bar{\theta}$ or $q_2 = \bar{\theta}$). Let

$$\theta^{++} = \frac{p_1^{++} - p_2}{q_1^{++} - \bar{\theta}} \tag{19}$$

be the indifferent consumer outside the gap (i.e., between purchasing $q_2^{-} = \bar{\theta}$ and q_1^{++}).

This implies that firm 1's payoff π_1 at the pricing stage (note that we use "π" to differentiate this payoff from the payoff at the quality stage) can be decomposed in three segments

$$\pi_1 = \pi_1^{a}(p_1^{+}, q_1^{+}) + \pi_1^{b}(p_1^{+}, p_2, q_1^{+}, \bar{\theta}) + \pi_1^{c}(p_1^{++}, p_2, q_1^{++}, \bar{\theta}) \tag{20}$$

where $\pi_1^{a}(p_1^{+}, q_1^{+})$ is the payoff from serving consumers $\theta \in [\theta_1^{*}, \theta_1^{**})$, $\pi_1^{b}(p_1^{+}, q_1^{+})$ is the payoff from serving $\theta \in [\theta_1^{**}, \tilde{\theta})$, and π_1^{c} is the "deviation" payoff from serving consumers $\theta \in [\theta^{++}, \bar{\theta}]$. We can express these payoffs as a function of prices and qualities:

$$\begin{aligned}
\pi_1^{a}(p_1^{+}, q_1^{+}) &= \int_{\frac{p_1^{+}}{q_1^{+}} - \frac{q_1^{+}}{4}}^{\frac{p_1^{+}}{q_1^{+}} + \frac{q_1^{+}}{4}} \left(\theta - \frac{p_1^{+}}{q_1^{+}} + \frac{q_1^{+}}{4}\right)^2 \frac{1}{\bar{\theta}} d\theta - \left(\frac{q_1^{+}}{2}\right)^2 \left(\frac{3q_1^{+}}{4} - \frac{p_1^{+}}{q_1^{+}}\right) \frac{1}{\bar{\theta}} && (21) \\
\pi_1^{b}(p_1^{+}, p_2, q_1^{+}, \bar{\theta}) &= \left(p_1^{+} - \frac{1}{2}(q_1^{+})^2\right) \left[\frac{p_2 - p_1^{+}}{\bar{\theta} - q_1^{+}} - \frac{p_1^{+}}{q_1^{+}} - \frac{q_1^{+}}{4}\right] \frac{1}{\bar{\theta}} && (22) \\
\pi_1^{c}(p_1^{++}, p_2, q_1^{++}, \bar{\theta}) &= \left(p_1^{++} - \frac{1}{2}(q_1^{++})^2\right) \left[\bar{\theta} - \frac{p_1^{++} - p_2}{q_1^{++} - \bar{\theta}}\right] \frac{1}{\bar{\theta}} && (23)
\end{aligned}$$

Similarly, firm 2's payoff at the pricing stage is given by:

$$\pi_2 = \left(p_2 - C(\bar{\theta})\right) \left[F(\theta^{++}) - F(\tilde{\theta})\right]$$

so using (18) and (19), we can obtain the first-order condition for an interior solution (i.e., $\theta^{++} < \bar{\theta}$) to firm 2's problem:

$$\frac{p_1^{++} - p_2}{q_1^{++} - \bar{\theta}} - \frac{p_2 - p_1^{+}}{\bar{\theta} - q_1^{+}} - \left(p_2 - \frac{1}{2}\bar{\theta}^2\right) \frac{1}{\bar{\theta} - q_1^{+}} = \left(p_2 - \frac{1}{2}\bar{\theta}^2\right) \frac{1}{q_1^{++} - \bar{\theta}} \tag{24}$$

It is clear from (24) that an interior solution requires $q_1^{++} < \sqrt{2}\bar{\theta}$. Had firm 1 deviated to $q_1^{++} \geq \sqrt{2}\bar{\theta}$, the relevant FOC would instead be (see Proposition 1):

$$\bar{\theta} - \frac{p_2^- - p_1^+}{\bar{\theta} - q_1^+} - \left(p_2 - \frac{1}{2}\bar{\theta}^2\right)\frac{1}{\bar{\theta} - q_1^+} = 0 \tag{25}$$

so that firm 2 will be setting p_2 as if firm 1 did not deviate at all (and $\theta^{++} = \bar{\theta}$). As q_1^{++} drops below $\sqrt{2}\bar{\theta}$, firm 2 follows a limit-pricing strategy, keeping $\theta^{++} = \bar{\theta}$. There is a point however, $q_1^{++} = 1.31\bar{\theta} \equiv \check{q}$, at which (24) begins to be binding. Since at this point, the price p_2 that solves (24) is strictly lower than the price p_2 that solves (25), we can rule out any deviation to $q_1^{++} \in [\check{q}, \hat{q})$ since it makes firm 1 *strictly* worse off (note that this drastic drop in profit will still apply for any sufficiently small t).

As firm 1 deviates to $q_1^{++} \in (\bar{\theta}, \check{q})$, two forces are at play: a direct effect and an strategic effect (for higher values of q_1^{++}, only the strategic effect was present). This can be seen by applying the envelop theorem to the profit expression (20) after plugging (21)-(23):

$$\frac{d\pi_1}{dq_1^{++}} = \frac{\partial \pi_1}{\partial q_1^{++}} + \frac{\partial \pi_1}{\partial p_2^-}\frac{\partial p_2}{\partial q_1^{++}}$$

where $\partial \pi_1 / \partial q_1^{++} < 0$ (the direct effect), $\partial \pi_1 / \partial p_2^- > 0$ and $\partial p_2^- / \partial q_1^{++} > 0$. The product of these last two terms constitutes the strategic effect, which is positive. It is not difficult to show, after some algebra, that the strategic effect *strictly* dominates the direct effect, i.e., $d\pi_1 / dq_1^{++} > 0$, for any $q_1^{++} \in (\bar{\theta}, \check{q})$. Consequently, no deviation outside the gap is profitable.

We have shown that any deviation to $q_1^{++} > \bar{\theta}$, while keeping q_1^+ at the equilibrium level in Proposition 1, makes firm 1 *strictly* worse off. Therefore, it remains to be seen whether allowing firm 1 to also deviate on q_1^+ alleviates matters in any way (in fact, firm 1 would have incentives to simultaneously increase q_1^+ somehow since the strategic effect within the gap is now weaker as p_2 is now less responsive). Such adjustment in q_1^+ indeed alleviates matters, but not enough. In the region $q_1^{++} \in (\check{q}, \hat{q})$, firm 1 continues making strict losses from the deviation since any such deviation reports no benefits outside the gap. And in the region $q_1^{++} \in (\bar{\theta}, \check{q})$, $d\pi_1 / dq_1^{++}$ continues to be positive for any $q_1^{++} \in (\bar{\theta}, \check{q})$.

Proof of Proposition 3 The proof is divided in two steps. The first step consists in showing that for $\mu \to 1$ it is profitable for firm 1 to deviate to carrying some $q > \bar{\theta}$ to better discriminate the non-shopper high types without attracting any additional shopper (for simplicity, we restrict attention to deviations to a single quality q). Because $\mu \to 1$, prices p_1^+ and p_2^- in Proposition 1 would remain unchanged had firm 1 not deviated to carry some quality $q > \bar{\theta}$. According to Proposition 1, these prices can be obtained from

(6) and (7). For the quadratic-uniform setting, these prices reduce to $p_1^+ = 3(q_1^+)^2/4$ and $p_2^- = q_1^+\bar{\theta}$, with $q_1^+ = (2-\sqrt{2})\bar{\theta}$.

Now, for it to be profitable and feasible to offer quality $q > \bar{\theta}$ to non-shopper high types at some price p (while holding p_1^+ fixed), the following profitability and participation conditions must hold, respectively

$$p - C(q) > p_1^+ - C(q_1^+)$$

and

$$\bar{\theta}q - p > \bar{\theta}q_1^+ - p_1^+$$

which lead to

$$\bar{\theta}q - C(q) > \bar{\theta}q_1^+ - C(q_1^+).$$

For the quadratic-setting, this reduces to $q < 2\bar{\theta} - q_1^+ = \sqrt{2}\bar{\theta}$. So, if firm 1 deviates to carry $q = \sqrt{2}\bar{\theta}$, it can barely attract the non-shopper highest type $\bar{\theta}$ for a price of $p_1^+ + C(q) - C(q_1^+) = p_1^+ + \bar{\theta}(q - q_1^+)$, reporting no extra profit on non-shoppers (and obviously nothing extra on shoppers).

Similarly, for quality $q > \bar{\theta}$ to be profitable and feasible to be offered to shopper high types for some price p (while holding firm 2's price offers fixed) the following two conditions must hold, respectively

$$p - C(q) > 0$$

and

$$\bar{\theta}q - p > \bar{\theta}^2 - p_2^-$$

which lead to

$$\bar{\theta}q - C(q) > \bar{\theta}^2 - p_2^-.$$

For the quadratic-setting (recall that $p_2^- = \bar{\theta}q_1^+$), this reduces to $q < \bar{\theta} + \sqrt{2q_1^+\bar{\theta} - \bar{\theta}^2} = \sqrt{2}\bar{\theta}$. So again, if firm 1 deviates to carry $q = \sqrt{2}\bar{\theta}$, it can barely attract the shopper highest type $\bar{\theta}$ for a price of $p = C(q)$, reporting no extra profit on shoppers but a strict loss on non-shoppers equal to

$$\frac{1-\mu}{2}(p_1^+ - q_1^+)(1 - F(\theta')) > 0,$$

where $\theta' = (C(q) - p_1^+)/(q - q_1^+) < \bar{\theta}$ is the non-shopper that is just indifferent between taking option (q_1^+, p_1^+) and option $(q, C(q))$.

Deviating to carrying $q = \sqrt{2}\bar{\theta}$ reports no extra profit to firm 1 when aiming this quality at the non-shoppers, by pricing it at $p = p_1^+ + C(q) - C(q_1^+)$, and a loss when aiming it at both the shoppers and non-shoppers, by pricing it at $p = C(q)$. Hence, from a standard continuity argument, there exists a quality $q^{NS} \in (\bar{\theta}, \sqrt{2}\bar{\theta})$ that leaves firm 1 just indifferent between aiming q^{NS} exclusively at the non-shoppers, by pricing it at $p' \in (p_1^+ + C(q^{NS}) - C(q_1^+), p_1^+ + \bar{\theta}(q^{NS} - q_1^+))$, and aiming it at both the shoppers and non-shoppers, by pricing it at $p'' \in (C(q^{NS}), \bar{\theta}q^{NS} + p_2^- - \bar{\theta}^2) < p'$.[29]

Having established that carrying q^{NS} is a profitable deviation for $\mu \to 1$, the second step of the proof is to show that firm 1 wants to deviate even further, to carry $q < q^{NS}$ to attract some shoppers high type (along with non-shoppers high type) with positive probability. When firm 1 deviates to carry $q < q^{NS}$, she anticipates two competitive responses from firm 2 in the pricing stage. The first is that firm 2 will price q more aggressively now. In the absence of non-shoppers, this (equilibrium) response would be to price q at cost $C(q)$. In the presence non-shoppers, however, the price competition for selling q will be in mixed strategies. The corresponding equilibrium is straightforward to characterize: Firm 1 will choose price $p_1(q) \in \{[\underline{p}(q), \bar{p}(q)], p^u(q)\}$ according to some cumulative distribution function $H_1(p; q)$, with $\underline{p} > C(q)$, $p^u > \bar{p}$ and $H_1(\bar{p}; q) < 1$ (i.e., firm 1 will put a mass $1 - H_1(\bar{p}; q) > 0$ at the upper bound p^u, where it only serves non-shoppers), while firm 2 will choose price $p_2(q) \in [\underline{p}(q), \bar{p}(q)]$ according to some (atomless) function $H_2(p; q)$.

The second response of firm 2 is a consequence of the first. Since firm 2 expects to loose some shoppers high type to firm 1 with positive probability, and hence, have fewer inframarginal shoppers buying q_2^-, firm 2 will also respond to $q < q^{NS}$ by lowering the price of q_2^- from its equilibrium level in Proposition 1.[30] So, in deciding whether to deviate to $q < q^{NS}$, firm 1 must trade off (i) the benefit of attracting some shoppers high type while extracting more from non-shoppers high type against (ii) the cost of increasing competition for shoppers in the gap. But at the margin, when $q = q^{NS}$, the latter effect is zero, because $H_1(\bar{p}; q^{NS}) = 0$ and $\underline{p}(q^{NS}) = \bar{p}(q^{NS})$.[31] Hence, the optimal deviation necessarily entails $q < q^{NS}$, where the marginal benefit of increasing q is equal to its marginal cost.

Proof of Lemma 3 Equilibrium profits at a symmetric equilibrium are fully determined by profits at the least generous menu, which is given by the monopoly solution.

[29] For example, $q^{NS} = 1.4\bar{\theta}$ for $\mu = 0.995$.

[30] Note that p_2^- also becomes random since it is decided simultaneosly with $p_2(q)$, but within a much tighter interval. We do not need to make any of this explicit for our proof.

[31] Firm 1's cost $\ell(q)$ of setting $q < q^{NS}$ can be expressed as

$$\ell(q) \sim \int_{\underline{p}(q)}^{\bar{p}(q)} \Delta(p; q) H_1(p; q) dH_2(p; q)$$

where $\Delta(p; q)$ is firm 2's inframarginal (high-type) shoppers lost to firm 1 when $p_1(q) < p_2(q)$. It follows that $\ell'(q^{NS}) = 0$.

Hence, as long as the first stage quality range does not constrain firms from implementing it, i.e., for $Q_i = [0, q^+]$ for $i = 1, 2$, with $q^+ \geq q_m\left(\bar{\theta}\right) = q_c\left(\bar{\theta}\right)$, expected equilibrium profits remain unchanged. For narrower product lines, it still holds that at any symmetric equilibrium with ordered menus expected profits are given by the highest profits that can be made out of the non–shoppers. However, these must be strictly below monopoly profits as firms do not carry all the qualities needed to extract monopoly profits from the non-shoppers.

Proof of Proposition 5 Consider deviations from a candidate symmetric equilibrium with $Q = [0, q^+]$, with $q^+ \geq q_m\left(\bar{\theta}\right) = q_c\left(\bar{\theta}\right)$ to $Q_i' = [q_i^-, q_i^+]$ with $q_i^- \geq 0$ and/or $q_i^+ \leq q^+$. Clearly, equilibrium profits would remain the same if the firm enlarges its product line, as it would not be constrained to implement the monopoly solution over the non-shoppers. Consider thus deviations to $q_i^- > 0$ and/or $q_i^+ \leq q^+$. At the least generous menu, firm i would be constrained to implement the optimal scheme over the non-shoppers. Hence, its profits must go down relative to the candidate equilibrium unless firm j plays a mass point at the least generous menu. However, this leads to a contradiction, as firm j would then be making lower profits than at the monopoly solution, which it can guarantee to itself by offering the MR solution.

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