A Model of Search with Price Discrimination*

Natalia Fabra
Universidad Carlos III and CEPR

Mar Reguant
Northwestern, CEPR and NBER

August 15, 2019

Abstract

We study the interaction between price discrimination and search in markets characterized by imperfect competition. For this purpose, we build a search model in which consumers differ in their willingness to pay as well as in their (private) search costs. Buyers' willingness to pay shape their willingness to search, thus affecting the intensity of competition among firms for the various consumers. The elasticity of the search cost distribution determines whether high or low valuation consumers are expected to search more, thus affecting whether firms compete relatively more to serve them. We show that a ban on price discrimination hurts the low valuation consumers as they end up searching more and paying higher prices. We provide a parametrization under which a ban on price discrimination is overall welfare improving.

Keywords: price discrimination, competition, search, bid solicitation.

1 Introduction

The interaction between price discrimination and search has become a major focus of recent competition policy cases, both in Europe as well as in the US.[1] Yet, while both search and price discrimination have long been studied as distinct issues, less is known about how they interact. In this paper we study the interaction between price discrimination and search in markets characterized by imperfect competition.

*Emails: natalia.fabra@uc3m.es and mar.reguant@northwestern.edu. Joao Montez, Jose Luis Moraga, David Roynane, Yossi Spiegel and seminar participants at ENTER Jamboree (Madrid), the Macci Conference (Mannheim), and the BGSE Consumer Search and Switching Costs Summer Workshop (Barcelona) provided useful comments. All errors remain our own. Fabra is grateful to the Economics Department of Northwestern University for their hospitality while working on this paper, and the Spanish Ministry of Education (Grant ECO2016-78632-P) for financial support.

[1]In the EU, in May 2015 the European Commission launched an antitrust investigation into e-commerce; its preliminary findings were published in September 2016 (European Commission, 2016). Also, in July 2016, the European Commission sent an Statement of Objections to Google concluding that Google had abused its dominant position by favouring its comparison shopping service in its search result pages. Competitive concerns over e-commerce have also arisen in the US. For instance, in April 2015 an e-commerce seller of posters was found guilty for price fixing through Amazon Marketplace.

Online markets provide an example in which such interaction becomes particularly meaningful. Search costs and the ability to price discriminate are key determinants of online prices for three main reasons. First, the internet provides a vast amount of consumer data which firms can use to tailor their pricing policies. Second, the internet allows firms to limit consumers' arbitrage opportunities, e.g. through the use of practices such as geo-blocking,[2] among others. And third, even though the internet has drastically reduced search frictions, there is ample evidence indicating that a large amount of online consumers do not engage in active search.[3]

In this paper we build an oligopoly model in which heterogeneous buyers engage in costly search. In particular, buyers differ in their willingness to pay as well as in their search costs. The former are assumed to be observable, while the latter are not. Hence, firms can practice third-degree price discrimination by conditioning prices on the buyers' valuation, but they cannot price discriminate on the basis of their search costs or their search history.[4] Differences in buyers' willingness to pay introduce ex-ante differences in their willingness to *engage in search* (extensive margin) as well as in their willingness to *search more* conditional on search (intensive margin). These two effects affect the intensity of rivalry among sellers when competing for either high or low valuation buyers, affecting the price differences among them.

A buyer's willingness to search depends on her valuation and on her search cost. Her valuation provides a signal about her gains from search, but her decision to engage in search conveys countervailing information about her search costs. Buyers with high valuations stand to gain more from search as they derive more utility from consuming the good ("gain-from-search effect"). However, precisely because they gain more from search, their decision to participate in search is less informative about their search costs ("signalling-through-participation effect"). The interaction between these two countervailing effects is key when addressing the main questions of this paper: how do consumers search and which prices do they pay (i) under price discrimination, (ii) under uniform pricing, and (iii) how do these two compare?

First, under price discrimination, we show that whether sellers compete more or less aggressively to serve the high or the low valuation consumers critically depends on the shape of the search cost distribution. More specifically, the relative importance of the "gains-from-search effect" versus the "signaling-through-participation effect" depends on the elasticity of the search cost distribution. When this elasticity is increasing (decreasing), buyers with high (low) valuation are expected to search less conditional on search, which in turn leads sellers to compete less aggressively to serve them. Intuitively, for a buyer to be willing to engage in search (extensive margin), her search cost must be below a high threshold; for her to be willing to search more than once (intensive margin), her search cost must be below a lower threshold. Hence, when the search cost distribution is more elastic at the upper (lower) deciles of the search cost distribution, an increase in the buyer's

[2] The European Commission (2016) documents that geo-blocking is widely used in e-commerce across the EU.

[3] For instance, in the market for online books, De los Santos et al. (2012) report that 74% of all consumers buy at the first store they visit without searching any further.

[4] In this respect, our paper differs from Armstrong and Vickers (2018), in which firms discriminate against captive consumers.

valuation affects more his likelihood to engage in search ("signaling-through-participation effect") than his search intensity ("gains-from-search effect"). In turn, this implies that sellers compete less (more) aggressively to serve buyers with high valuation. For iso-elastic functions (e.g. when search costs are uniformly distributed, with no mass at zero search costs), the two effects cancel out. This implies that the intensity of competition for high or low valuation consumers is equal, so that price differences simply reflect differences in the valuations.[5]

Our analysis thus suggests that the elasticity of the search cost distribution plays a key role in determining the intensity of competition for the various buyers, and thus the extent of price discrimination.[6] Several commonly-used distributions, such as the exponential or the Pareto distribution, depict monotonically decreasing elasticities, thus implying that sellers compete more strongly to serve buyers with high valuations. This narrows down the price differences between high and low valuation customers. However, other distributions have non-monotonic elasticities (e.g. the Normal or the Gamma distributions), thus implying that competition is non-monotonic in their valuations.[7] Interestingly, with normally distributed search costs, sellers compete strongly for buyers with very low or very high valuations, but not so much for the remaining customers. The latter do not have small enough valuations to benefit from signaling a low search cost when they engage in search, and their valuations are not high enough to benefit from high gains from search.

Second, under uniform pricing firms charge the same prices to all buyers, just as if they were facing the average consumer. Hence, differences in the prices paid by the various consumers simply reflect differences in their search intensities. In particular, since conditional on search, high valuation consumers search less, on average they end up paying higher prices, even though the participate in search more often. An increase in the proportion of high valuation consumers leads to higher prices for all consumers as it reduces the expected search intensity of the average buyer.

Last, in order to shed light on the distributional effects of price discrimination, we compare equilibrium pricing and market outcomes under uniform prices versus price discrimination. This issue has become particularly relevant with the widespread of internet. Whether online retailers should be banned from price discriminating (e.g. by forcing them to relax the constraints on consumers' arbitrage, such as geo-blocking), or whether they should be banned from using most-favoured-nation clauses (MFNs) that lead to price uniformity, are contentious issues in the policy arena.[8] Beyond the welfare implications of price discrimination, our analysis reveals that the

[5]This discussion refers to the ranking of prices conditional on search, not to buyers' expected utility prior to search. Since high valuation buyers are relatively more likely to engage in search, their expected utilities tend to be higher all else equal.

[6]To be sure, estimating the distribution is not straightforward. See Hortaçsu and Syverson (2004), Hong and Shum (2006), Moraga and Wildenbeest (2008) and De los Santos et al. (2012) for papers that perform this task in various contexts.

[7]For instance, in the case of online books, Hong and Shum (2006) and De los Santos et al. (2012) find that the distribution of search costs depicts non-monotonic elasticities.

[8]Fort instance, in January 2015, the UK Competition and Markets Authority opened a call for information on the commercial use of consumer data. Furthermore, they have discouraged the use of wide MFNs or price parity provisions in areas such as motor insurance. As already mentioned, the European Commission has also raised concerns about residence-based price discrimination in several occasions.

effects are likely to differ across consumer types. While it is well known that price discrimination has distributive effects, the novelty of our analysis is to point out that a ban on price discrimination does not necessarily benefit buyers with low valuations. Indeed, under commonly used distributions (e.g. the uniform distribution with a mass of shoppers, or the Normal distribution), low valuation consumers tend to search more under uniform pricing and they also tend to pay higher prices. In turn, this implies that buyers with low valuation are less likely to engage in search with uniform prices than under price discrimination. In contrast, high valuation consumers free-ride on the more intense search activity of the low valuation consumers. They search less under uniform pricing and yet, at least when the mass of high valuation consumers is not too high, they end up paying lower prices than under price discrimination.

There are several trade-offs involved in the effects of banning price discrimination on total consumers surplus, profits and total welfare. These trade-offs arise because of the countervailing effects on competition, participation and search intensity across consumer groups. While it is not possible to provide a general welfare ranking between discrimination and uniform pricing, numerical simulations show that a ban on price discrimination may increase overall welfare because the reduction in the surplus of the low valuation consumers is more than compensated by the increase in the surplus of the high valuation consumers.

Related literature Since the seminal work of Diamond (1971), a broad literature has emerged analyzing the impact of consumers' search costs on market outcomes. However, to the best of our knowledge, all existing papers in the search literature assume *ex-ante* identical consumers, i.e., they all have unit demands and equal valuations of the good, and either their search costs are also equal, or they are unobserved by the sellers. Hence, these papers are not well equipped to analyze the interaction between search costs and price discrimination based on observable consumer characteristics. It is important to note that, search costs typically generate equilibrium price dispersion (Varian, 1980; Burdett and Judd, 1983; Stahl, 1989; Janssen and Moraga, 2004, among others), which in turn implies that consumers who search more end up paying lower prices. However, such price differences are not due to price discrimination *per se*, i.e., to sellers charging different prices to different buyers, but rather reflect differences in search intensity.

Some recent papers have started to analyze the interaction between search and third degree price discrimination on other dimensions.[9] In a model of sequential search, Janssen and Rushidi (2018) analyze price discrimination by wholesale firms (yet, at the downstream market, firms charge equal prices to all customers). Charging different prices to the various retailers creates price dispersion in the downstream market, which in turn induces more search and stronger retail market competition, to the benefit of wholesalers. Also, some recent papers allow for discrimination based on consumers' search costs (see Armstrong and Vickers (2018) for a theoretical analysis; and Gugler et al. (2019)

[9]Other papers on search and price discrimination include Fabra and Montero (2019) on second-degree price discrimination, Petrikaite (2018) on price discrimination through obfuscation, and Armstrong and Zhou (2016) on price discrimination through search deterrence. However, the sources and nature of price discrimination in those analyses markedly differ from the one in the current paper.

for an empirical analysis). This form of price discrimination is different from the one analyzed in this paper, as we assume that sellers do not observe consumers search costs or search history.

Another aspect further distinguishes our paper from most of the existing search literature: instead of taking search as given, we allow buyers to endogenously choose whether to engage in search or not. Endogenous participation is a key driver of price discrimination because the decision to participate in search conveys different information regarding search costs depending on the buyer's valuation.[10] A notable exception is Moraga et al. (2017b), who develop a sequential search model with differentiated products and endogenous search.[11] They show that a reduction in search costs can give rise to price increases as changes in the pool of consumers who engage in search make the market demand more inelastic. Our analysis shares some of the driving forces in Moraga et al. (2017b), but relies on different assumptions and pursues different objectives. In particular, in Moraga et al. (2017b) firms have no ex-ante information about individual buyers' characteristics, leaving no scope for price discrimination, or for a comparison between price discrimination and uniform pricing.

The recent literature on bidder solicitation also endogenizes search decisions. In particular, Lauermann and Wolinsky (2017) allow buyers to choose how many prices to solicit from a set of potential sellers at a *solicitation cost*, similar to the *search cost* in our paper (see also Lauermann and Wolinsky, 2016). The two analyses differ in one key aspect: in Lauermann and Wolinsky (2017), the buyer is privately informed about the value of the good (common-value setting); in our paper, she is privately informed about the solicitation or search cost (private-value setting). Despite these differences, both setups share an important ingredient: the buyer's participation decision (in solicitation or in search) provides a signal about the buyer's private information. In Lauermann and Wolinsky (2017), sellers are more likely to be solicited when the value of the good is low; hence, the solicitation effect softens competition. In our model, the signaling-through-participation effect enhances competition more for low valuation than for high valuation buyers as the decision to engage in search by the former is more informative about her search costs being low.

The remainder of the paper is structured as follows. In section 2, we construct and solve the model when price discrimination is allowed. We first characterize sellers' pricing decisions (section 3.1) and buyers' search behavior (section 3.2), and then prove existence of the equilibrium (section 3.3). Comparative statics of equilibrium prices with respect to buyers' valuations are performed in section 3.4. In section 4, we characterize the equilibrium when price discrimination is banned, while in section 5 we compare the equilibria under price discrimination and uniform pricing. Section 6 of the paper concludes. All proofs are included in the appendix.

[10] This driver is absent in the majority of the search models as these assume that buyers search at least once (either because search costs are sufficiently low, or because the first quote is for free).

[11] See also Moraga et al. (2017a) for the simultaneous version of the game, in which they study the impact of an increase in the number of sellers.

2 Model Description

Two symmetric sellers compete to sell an homogeneous good. A buyer's valuation for the good is denoted $v \in [\underline{v}, \overline{v}]$. Buyers decide whether to *search* to find out the prices of the competing sellers.[12] If a buyer does not search, her reservation utility is normalized to zero. The marginal cost of producing the good is normalized to zero.

Buyers incur a search cost c to observe the price of each seller.[13] Each buyer's search cost is private information, but it is common knowledge that c is drawn from the cumulative distribution function $G(c)$, with density $g(c) > 0$, in the interval $[\underline{c}, \overline{c})$, with $\underline{c} \leq 0$.[14] We assume $\underline{c} \leq \underline{v} \leq \overline{v} < \overline{c}$ to focus on the interesting cases in which buyers do not engage in search with some probability.[15]

The timing of the game is as follows. First, each buyer observes her realized search cost and takes her search decision: not to search, search once (picking one of the two sellers at random), or search twice. Sellers do not observe the buyers' search costs nor their search behavior. Second, each seller chooses a price b and each buyer buys from the seller that offered the lowest price among the ones she observes (ties are broken randomly).

We examine (symmetric) equilibria in which buyers maximize their utility given her correct beliefs about the sellers' pricing behavior, and the sellers maximize their profits given their correct beliefs about buyers' search behavior.

We consider solve two variants of the game. Under discrimination, sellers can condition their pricing strategies on each buyer's valuation v and are thus allowed to offer different prices across consumers. To the contrary, under no discrimination, sellers are forced to charge uniform prices to all consumer types.

3 Price Discrimination

In this section we assume that sellers are allowed to charge different prices to consumers as a function of their valuations. Hence, without loss of generality, we characterize the equilibrium when sellers face a single buyer with valuation v.

[12] Alternatively, one could assume that a buyer who does not search can buy the good at a default price. This would apply to several settings, e.g. in energy markets, consumers typically have access to energy at a regulated price, and consumers also have the option to search for competing retailers. The results of the model would remain the same.

[13] We could introduce scale economies (or diseconomies) in search by assuming that the second quote costs δc, with $\delta < 1$ ($\delta > 1$). Our main results would remain qualitatively unchanged.

[14] It is important to note that the two sources of buyer heterogeneity (valuations and search costs) are not isomorphic. For instance, conditional on search (for given prices), the buyer's valuation does not affect her incentives to search more as she enjoys the good regardless of how much she searches. Instead, the buyer's search cost affects how much she wants to search. See section 3.2 for more details.

[15] Unlike other models in the search literature (e.g. Varian's model of sales), shoppers are not essential in our model. Indeed, our model has $\underline{c} = 0$ (i.e., there are no shoppers) as a particular case. However, the mass of shoppers may have an impact on the nature of the equilibria in interesting dimensions (see section 3.4).

3.1 Pricing decisions

We start by analyzing sellers' pricing behavior given their expectations about the buyer's search behavior. Consider a seller's pricing decision. The seller does not observe the buyer's search behavior but believes that, *conditionally on search*, the buyer has searched once with probability ρ. In equilibrium, this expectation must turn out to be correct. To the extent that the buyer's valuation v conveys different information about the her search behavior, sellers might hold different beliefs about ρ depending on v. In this section, we take ρ as a parameter, and we will endogenize it later in section 3.3.

Our first result shows that in equilibrium, conditional on search, some buyers types search once while others search twice. Hence, each seller does not know with certainty whether the buyer will not observe his price, whether he will be a monopolist over the buyer, or whether he will be competing with the other seller. The only information that sellers can infer is that the buyer's search cost must be low enough for her to be willing to search at least once.

Lemma 1 *In a SPNE, if the buyer engages in search, the probability that she searches once is* $\rho \in (0, 1)$*. As a consequence, there does not exist pure-strategy equilibria in prices.*

The intuition for this result is well known (Burdett and Judd, 1983). On the one hand, if the buyer searches once with certainty ($\rho = 1$), the seller charges the monopoly price. However, as this would leave no surplus for the buyer, she would not search in the first place (Diamond's paradox). On the other hand, if the buyer searches twice with certainty ($\rho = 0$), sellers engage in Bertrand competition. Since all sellers would then price at marginal costs, the buyer would have incentives to search only once. As a consequence, neither the monopoly price nor the competitive price can be sustained in a SPNE.[16,17]

More generally, $\rho \in (0, 1)$ rules out the existence of pure strategy equilibria regardless of the buyer's valuation:[18] starting from any arbitrary price pair, sellers would like to undercut each other until prices are so low that it becomes optimal for a seller to price at the consumer's maximum willingness to pay for the good. However, if one seller is pricing at that level, it becomes profitable for the other seller to slightly undercut it. Therefore, the equilibrium has to be in mixed strategies and, using the same Bertrand argument, it must be atomless.[19]

To distinguish the price offered by the seller from the price actually chosen by the buyer, we refer the former as a *quote*. Let sellers use the (symmetric) quote distribution $F(b)$. Conditional

[16] Note that this result would remain unchanged in the case of $N > 2$ firms.

[17] If the first quote is free (as in Burdett and Judd, 1983), there always exists an equilibrium where all firms charge the monopoly price.

[18] This result is reminiscent of Janssen and Rasmusen (2002) in which with an exogenously given probability, rival firms may be inactive. In contrast, in this paper we endogenize this probability through the analysis of consumers' search (see next section).

[19] This is in contrast to Lauermann and Wolinsky (2016), in which the common value assumption gives rise to an atom in bidders' strategies. This atom implies a failure of competition to aggregate information even when search costs are very low and competition is strong.

on the buyer's decision to search, a seller's expected profits form pricing at b are given by

$$\pi(b) = b\left[\rho/2 + (1-\rho)(1-F(b))\right].$$

A seller's mixed strategy strikes a balance between two opposing forces. On the one hand, sellers benefit from charging a high price to a buyer who has only searched once. This event occurs with probability $\rho/2$ (recall that both sellers are equally likely to be picked by the buyer).[20] On the other hand, sellers also benefit from charging a low price, as it is thus more likely to be chosen by a buyer who has searched twice. This event occurs with probability $(1-\rho)(1-F(b))$. Proposition 1 below characterizes the equilibrium quote distribution.[21]

Proposition 1 *Assume $\rho \in (0,1)$. There is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by*

$$F(b) = 1 - \frac{1}{2}\frac{\rho}{1-\rho}\frac{v-b}{b}$$

with compact support $b \in \left[\frac{v\rho}{2-\rho}, v\right]$.

Interestingly, the buyer's valuation v enters directly into the sellers' pricing strategy but also indirectly through ρ, i.e., the sellers' expectation of the buyer's search strategy. Indeed, an increase in ρ shifts the whole quote distribution to the right in a FOSD sense, also increasing the lower bound of the quote support. Intuitively, an increase in ρ implies that sellers price less aggressively, leading to higher expected quotes.[22] In other words, a buyer who is expected to search more (conditional on search) observes lower quotes, regardless of her actual search intensity (which also depends on her realized search cost).

Clearly, sellers never quote prices above the buyer's maximum willingness to pay. Therefore, conditionally on search, the buyer always buys from one of the sellers. In particular, if a buyer observes only one quote (which conditionally on search, occurs with probability ρ), she simply pays that quote, which in expectation equals

$$E[b] = \int_{\underline{b}}^{v} b\,dF(b).$$

However, if she observes two quotes (with probability $1-\rho$), she pays the minimum of the two, which in expectation equals

$$E[\min\{b_1, b_2\}] = \int_{\underline{b}}^{v} 2b(1-F(b))\,dF(b).$$

[20] Note that profits are represented conditional on a buyer searching. Results are unaffected if we instead compute expected profits conditional on the firm having received a quote request, i.e., if we re-scale profits by $1/(1-\rho/2)$.

[21] Hong and Shum (2006) and Moraga and Wildenbeest (2008) derive the same distribution for the case with $N > 2$ firms, but treat the ρ parameters as exogenously given. Also, they only consider consumers with unit demands.

[22] The fact that ρ shrinks the quote support does not mean that dispersion is reduced because ρ also affects the shape of the distribution. Indeed, an increase in ρ has a non-monotonic effect on the dispersion of quotes: it first increases and it then decreases as ρ goes up.

Putting both pieces together and using the quote distribution characterized in Proposition 1, the next lemma computes the expected price paid by a buyer conditional on search.

Lemma 2 *Conditional on search, the expected price paid by the buyer is* $v\rho \in (0, v)$ *.*

Importantly, the Lemma above implies that expected prices conditional on search are increasing in ρ. The reason is two-fold. First, there is a simple *probability effect*: the higher ρ, the less likely it is that the buyer can compare the two quotes and choose the lowest one. And second, there is a *competition effect*: the higher ρ, the weaker is the competition between the sellers and hence the higher the expected quotes. Thus, expected search behavior has a direct translation on how total surplus (gross of search costs) is distributed: conditional on search, sellers make profits $v\rho$, and the buyer obtains gross utility $v(1-\rho)$.[23] Put differently, ρ is a measure of the sellers' market power that allows them to extract a great fraction of the consumer's surplus.

3.2 Search decisions

Once the buyer has observed her realized search cost c, she has to decide whether to engage in search and if so, whether to search once or twice. Her utility from searching once or twice is respectively given by u_1 and u_2,

$$\begin{aligned} u_1 &= (v - E[b]) - c \\ u_2 &= (v - E[\min\{b_1, b_2\}]) - 2c. \end{aligned}$$

The next Proposition characterizes the buyer's optimal search strategy.

Proposition 2 *The buyer's optimal search strategy follows a cut-off rule: for given* $\rho \in (0,1)$ *, there exist* $0 < c_1(v) < c_0(v) < v$ *such that the buyer does not search if* $c > c_0(v)$ *, she searches once if* $c \in (c_1(v), c_0(v)]$ *or she searches twice otherwise.*

For the buyer to find it optimal to engage in search, her utility from doing so must be non-negative, i.e., the expected gross utility from search must exceed her search cost. This amounts to

$$c \leq c_0 = v - E[b].$$

Using the distribution of price quotes characterized in Proposition 1, c_0 can be expressed as

$$c_0 = v\left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\frac{2-\rho}{\rho}\right).$$

[23] Search intensity also affects price dispersion. Indeed, the coefficient of variation (measured as the ratio between the standard deviation and the mean of prices, see Sorensen (2000), among others) is monotonically decreasing in ρ. Hence, one should expect lower price dispersion in markets with higher prices –a result that is reminiscent of Stigler's seminal contribution despite the fact that he assumed the quote distribution to be exogenously given, i.e., independent of the intensity of search.

All else equal, $c_0(v)$ is increasing in v: higher valuation consumers are more likely to engage in search because they enjoy more from consuming the good. To the contrary, $c_0(v)$ is decreasing in ρ: a higher ρ reflects higher expected prices, making search less attractive. On one extreme, if $\rho = 0$, expected prices are equal to (zero) marginal costs; hence, the buyer engages in search whenever search costs do not exceed her valuation, $c \le c_0 = v$. On the other extreme, if $\rho = 1$, expected prices are equal to the buyer's valuation. Hence, the buyer engages in search only if it is free, $c \le c_0 = 0$. In equilibrium, since $\rho \in (0,1)$ (Lemma 1), $c_0 \in (0, v)$.

Consider now the decision to search twice: for it to be optimal, the buyer's search cost has to be below the expected savings from observing two rather than just one quote. This amounts to

$$c \le c_1 = E[b] - E[\min\{b_1, b_2\}].$$

The threshold c_1 does not depend directly on v: the buyer's valuation has no direct effect on the propensity to search twice rather than once as the buyer enjoys the good in either case. However, c_1 depends indirectly on v through the impact of sellers' pricing behavior. Indeed, using Proposition 1, c_1 can be expressed as

$$c_1 = v - \frac{c_0(v)}{1-\rho}$$

Overall, c_1 increases in v, but less than c_0. Unlike c_0, the threshold c_1 is non-monotonic in ρ given that ρ affects expected prices both when the buyer searches once as well as when she searches twice, with both effects pushing c_1 in opposite directions.[24]

3.3 Equilibrium characterization

In equilibrium, the buyer's beliefs about the distribution of quotes in the market must be consistent with sellers' actual pricing behavior. Likewise, conditional on search, sellers' beliefs about the buyer's search strategy must be consistent with her actual search behavior. Thus, in equilibrium, the following condition must satisfied:

$$\rho^* = 1 - G\left(c_1(\rho^*) \middle| c \le c_0(\rho^*)\right).$$

Importantly, since the buyer only finds it optimal to search when her search cost realization is sufficiently low, her participation decision signals that his search cost is below c_0. The equilibrium condition incorporates this, as the distribution that sellers use to compute the buyer's expected search behavior is truncated at c_0.[25] More succinctly, the above expression can be re-written as

$$\rho^* = 1 - \frac{G(c_1^*)}{G(c_0^*)} \in (0,1). \tag{1}$$

[24] In particular, c_1 first increases and then decreases in ρ. Furthermore, $\lim_{\rho\to 0} c_1 = \lim_{\rho \to 1} c_1 = 0$. See the appendix for details.

[25] If the first quote is for free, as assumed in various papers of the search literature, participation is not an issue. In this case, the equilibrium condition is simply $\rho^* = 1 - G(c_1^*)$. It is easy to see that in this case a unique equilibrium always exists.

Together with our previous results, the solution to equation (1) completes the characterization of the Subgame Perfect Nash Equilibrium (SPNE). The following Proposition guarantees that there always exists a solution to equation (1), which is interior (in line with Lemma 1). Furthermore, the Proposition also identifies sufficient conditions for this solution to be unique.

Proposition 3 *There exists a symmetric SPNE in which sellers price as stated in Proposition 1 and buyers search as stated in Proposition 2. Conditional on search, the probability that the buyer searches once is given by the solution to* (1)*. A sufficient condition for the equilibrium to be unique is that the elasticity of the search cost distribution G,*[26] $\varepsilon(c) \equiv cg(c)/G(c)$, *is increasing in c.*

The equilibrium is guaranteed to be unique for G whose elasticity is decreasing in c. This guarantees that the right hand side of equation (1) is everywhere decreasing in ρ and hence crosses the 45-degree line only once. If the elasticity is not everywhere increasing, a sufficient condition for uniqueness is that G is not too concave.[27] The family of distribution functions for which the solution is unique is very broad, including the uniform, normal, and exponential distributions, plus all log-convex distributions, among others. In the rest of the paper, we assume that the search cost distribution is always such that the equilibrium is unique.

3.4 How do prices depend on the buyer's valuation?

We are now ready to assess whether sellers compete more or less aggressively when selling to high or low valuation buyers.

Proposition 4 *In equilibrium, (i)* $\rho^*(v)$ *is increasing in* v *if* $\varepsilon(c_0^*) > \varepsilon(c_1^*)$*; (ii)* $\rho^*(v)$ *is decreasing in* v *if* $\varepsilon(c_0^*) < \varepsilon(c_1^*)$*; and (iii)* $\rho^*(v)$ *is independent of* v *if* $\varepsilon(c_0^*) = \varepsilon(c_1^*)$.

Sellers compete more aggressively for buyers who are expected to search more. But, is it always the case that, conditional on search, sellers expect high valuation buyers to search more? The answer is no. To understand why, let us decompose a buyer's willingness to search in two components: the *gains from search* and the *costs of search*. On the one hand, the gains from search are higher for buyers with higher valuation given that any potential price savings achieved through search are proportional to v. We refer to this as the "gains-from-search effect". On the other hand, even if all buyers have ex-ante equal expected search costs, their decisions to engage in search convey different information regarding their search costs. Precisely because buyers with low valuation stand to gain less from search, sellers expect those buyers with low valuation who engage in search to have relatively lower search costs. We refer to this as the "signalling-through-participation effect". Thus, whether sellers expect buyers with low or high valuations to search

[26] Note that the elasticity can also be expressed as $\varepsilon(c) \equiv cr(c)$, where $r = g/G$ is the reverse hazard rate. The elasticity is decreasing in c if $\partial r/\partial c = -r/c$, i.e., the reverse hazard rate has to be sufficiently decreasing in c. For G log-convex, the elasticity is everywhere increasing.

[27] As shown in the Appendix, this guarantees that the slope of the schedule $1 - G(c_1)/G(c_0)$ is either negative or, if positively sloped, its slope is never above 1.

more, conditional on search, critically depends on the interplay between these two countervailing effects.[28]

The "gains-from-search" and the "signalling-through-participation" effects work in opposite directions. Whether one effect or the other dominates, depends on the shape of G. In particular, it depends on the elasticity of the search cost distribution.[29] Suppose that the elasticity of $G(c)$ at c_0^* is greater (lower) than at c_1^*. Hence, from the sellers' point of view, an increase in v increases the probability that the buyer asks for two quotes, $G(c_1^*)$, less (more) than it increases the probability that the buyer engages in search, $G(c_0^*)$. Hence, the conditional probability that the buyer has searched only once, ρ^*, increases (decreases) in v. In sum, if the elasticity of the search cost distribution is higher (lower) at c_0^* than at c_1^*, sellers expect that, conditional on search, buyers with high valuation are less likely to search twice than buyers with low valuation. Hence, sellers compete less (more) aggressively to serve the higher (low) valuation buyers. If the elasticity is the same at these two thresholds, the "gains-from-search" and the "signalling-through-participation" effects cancel out. Hence, sellers expect all buyers to search with the same intensity, regardless of their valuation.[30]

The shape of the search cost distribution ultimately determines whether $\varepsilon(c_1^*)$ is higher or smaller than $\varepsilon(c_0^*)$, and thus the comparative statics of ρ^* with respect to v. A sufficient condition for ρ^* to be increasing, decreasing or constant in v is that the elasticity of $G(c)$ is either increasing, decreasing or constant, respectively. There are several distribution functions with monotone elasticities. If search costs are uniformly distributed with $\underline{c} < 0$ (i.e., some buyers enjoy searching), the elasticity is increasing in c in the interior range (for $c > 0$), so that sellers compete more for low valuation buyers. However, in the absence of shoppers, i.e., $\underline{c} = 0$, the elasticity is constant so that all consumers pay the same price. In contrast, under the exponential or the Pareto distribution, elasticities decrease monotonically in c, implying that sellers compete more to serve high valuation consumers. For many commonly used distributions (e.g. the Normal distribution), the elasticity is non-monotonic. However, for v such that at both c_1^* and c_0^* the elasticity is either increasing or decreasing, the same comparative static results as above apply.

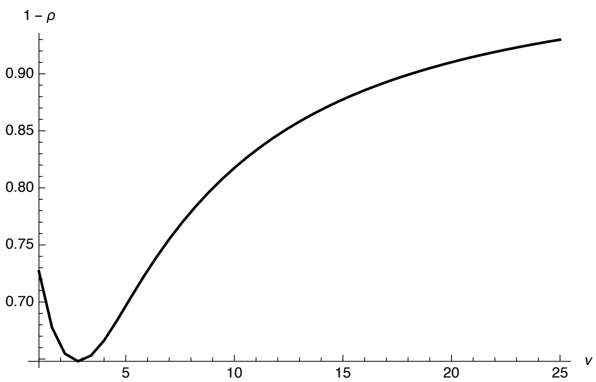
Figure 1 provides graphical examples of the equilibrium search intensity $(1-\rho^*)$ as a function of v under alternative distributional assumptions. One can see that the search intensity follows a non-monotonic pattern in the case of the Normal distribution (panel a), being lowest for medium values of v. The probability $1-\rho^*$ is always increasing in v under the exponential distribution (panel b), due to the elasticity being everywhere decreasing. Finally, for the case of the uniform distribution, $1-\rho^*$ is decreasing in v in the presence of shoppers (panel c), but it is constant

[28] If the buyer can observe the first quote for free, the equilibrium comparative statics show that $\partial\rho^*/\partial v = -(\partial c_1^*/\partial v)g(c_1^*) < 0$, i.e., high valuation buyers always search more and firms always compete more fiercely to serve them. Thus, the fact that the search participation decisions are endogenous is crucial to assess the comparative statics of prices as a function of consumers' valuations.

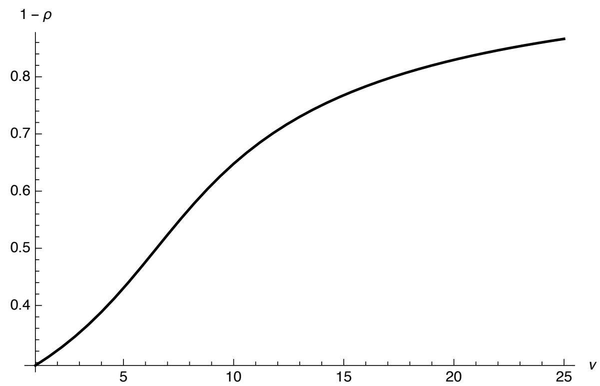
[29] There are several log-concave distribution functions for which $\varepsilon(c)$ is decreasing, e.g. the Exponential function $G(c) = \left(1 - e^{-c}\right)$ or the Pareto distribution $G(c) = 1 - \frac{1}{(c+1)^{\alpha}}$. The family of functions $G(c) = c^{\varepsilon}$ with $\underline{c} = 0$ have constant elasticity.

[30] This discussion is reminiscent of the literature in Public Finance that deals with the elasticity of earnings with respect to the tax rate over the distribution of income. See Saez (2001).

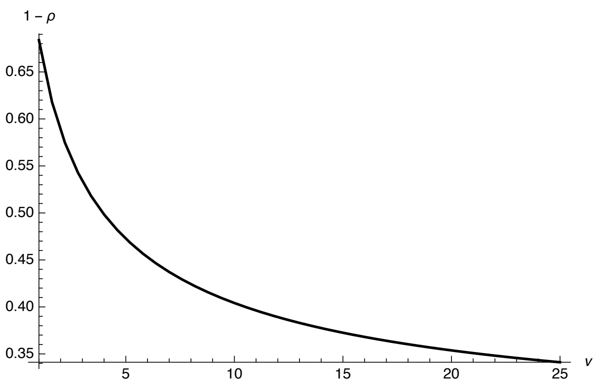
Figure 1: Expected search for second quote $(1-\rho^{*})$ as a function of valuation v



(a) Non-monotonic search intensity for Normal distribution, $\{\mu = 0, \sigma = 1\}$



(b) Increasing search intensity for Exponential distribution, $\{\beta = 1\}$



(c) Decreasing search intensity for Uniform distribution, $\{\underline{c} = -1, \overline{c} = 25\}$

(d) Flat search intensity for Uniform distribution, $\{\underline{c} = 0, \overline{c} = 25\}$

Notes: Normal distribution as $N(\mu, \sigma)$. Exponential distribution parametrized as $CDF(c, \beta) = 1 - e^{\frac{c}{\beta}}$. Uniform distribution in range $U \sim [\underline{c}, \overline{c}]$.

otherwise (panel d). Indeed, when search costs are uniformly distributed between -1 and 25, i.e., with a 20% mass of shoppers, consumers with high v are expected to search less conditional on search. Instead, when search costs are uniformly distributed between 0 and 25 all consumers are expected to have the same search intensity, conditional on search. This illustrates that the presence of shoppers can have a strong impact on the search behaviour of the other buyers through the price impacts.

The results shown in Proposition 4 affect the ranking of the expected prices paid by high or low valuation consumers. In particular, conditional on search, the expected price paid by a consumer with valuation v is $v\rho^{*}(v)$ (Lemma 2). Hence, a marginal increase in v implies that the price conditional on search changes by $\rho^{*}(v) + v\,(\partial\rho^{*}/\partial v)$. Whereas the first term is unambiguously positive, the sign of the second term depends on the shape of G, as shown above. If the elasticity of the search cost distribution is either increasing or constant, then higher valuation consumers unambiguously pay higher expected prices conditional on search. However, if the elasticity is decreasing and the valuations of the two types are sufficiently apart, the competition effect can outweigh the valuation effect, thus implying that high valuation consumers may end up paying relatively lower prices.

Figure 2 provides graphical examples of the resulting expected equilibrium prices as a function of v under alternative distributional assumptions. As shown in panels (c) and (b), expected prices follow a non-monotonic pattern in the case of the Normal and exponential distributions, being highest for medium values of v. Even though firms could charge higher prices to higher valuation consumers, they also tend to compete more to attract them as they are expected to search more. Under the uniform distribution, for which search intensity was either decreasing or flat, both effects work in the same direction leading to expected prices that are always increasing with the willingness to pay, as shown in panels (c) and (d).

We can use the model to shed light on a related question: the likelihood with which buyers with different valuations engage in search at all versus taking their outside option, given by $G\left(c_0^*\right)$. Taking the derivative with respect to v,

$$\frac{\partial G\left(c_0^*\right)}{\partial v} = g\left(c_0^*\right)\left(\frac{c_0^*}{v} + \frac{\partial c_0^*}{\partial \rho^*}\frac{\partial \rho^*}{\partial v}\right). \tag{2}$$

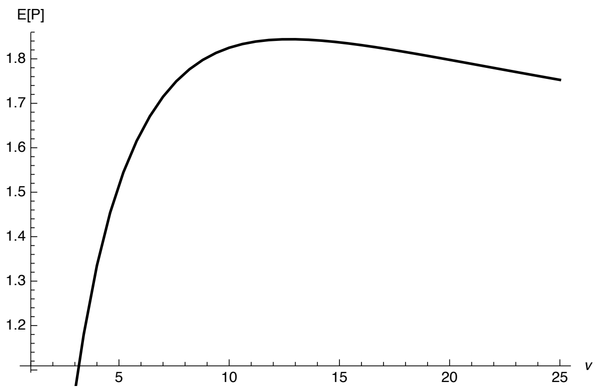
The likelihood of search is increasing in c_0^*, which in turn is increasing in v and decreasing in ρ^*. Thus, for given ρ, buyers with high valuations participate in search more often. However, c_0^* also depends on v through ρ^*. If ρ^* is non-decreasing in v (which according to Proposition 4 occurs if $\varepsilon\left(c_0^*\right) \geq \varepsilon\left(c_1^*\right)$), then equation (2) is unambiguously positive: buyers with high valuation engage in search more often both because they gain more from search, but also because sellers compete more strongly to serve them.

Matters are not as clear when $\varepsilon\left(c_0^*\right) < \varepsilon\left(c_i^*\right)$. If ρ^* is decreasing in v, the sign of equation (2) is *a priori* ambiguous (the first term in parenthesis is positive while the second one is negative). However, a simple logic shows that equation (2) cannot be negative: a necessary condition for low valuation buyers to obtain lower prices conditional on search is that their participation decisions signal sufficiently low search costs; for this to be the case, small buyers must engage in search less often. Hence, $\partial \rho^*/\partial v$ positive (i.e., conditional on search, low valuation buyers obtain lower prices) would contradict $\partial G\left(c_0^*\right)/\partial v$ negative (i.e., low valuation buyers engage in search more often), and *viceversa*.. This logic suggests that $\partial G\left(c_0^*\right)/\partial v$ must be positive, regardless of the shape of G.

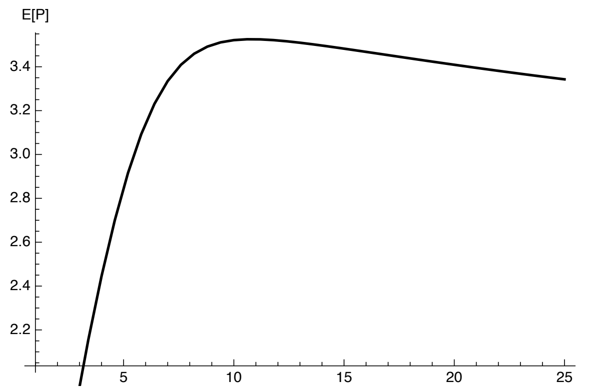
4 Uniform Prices

In this section we characterize equilibrium prices when sellers have to quote the same prices to all the buyers, irrespective of their valuation. For the sake of simplicity, we now assume that there are two types of buyers. In particular, there is a fraction α (resp. $(1-\alpha)$) of consumers have a high

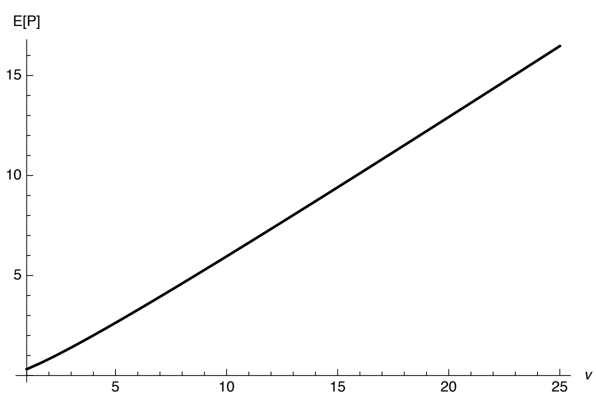
Figure 2: Expected prices (conditional on search) $v\rho^*$ as a function of valuation v



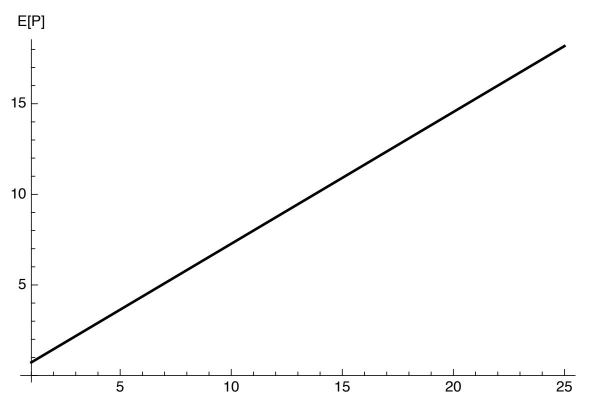
(a) Non-monotonic expected prices (conditional on search) for Normal distribution, $\{\mu = 0, \sigma = 1\}$



(b) Non-monotonic expected prices (conditional on search) for Exponential distribution, $\{\beta = 1\}$



(c) Increasing expected prices (conditional on search) for Uniform distribution, $\{\underline{c} = -1, \overline{c} = 25\}$



(d) Increasing (conditional on search) for Uniform distribution, $\{\underline{c} = 0, \overline{c} = 25\}$

Notes: Normal distribution as $N(\mu, \sigma)$. Exponential distribution parametrized as $CDF(c, \beta) = 1 - e^{\frac{c}{\beta}}$. Uniform distribution in range $U \sim [\underline{c}, \overline{c}]$.

valuation for the good, $v = \overline{v}$ (resp. $v = \underline{v} < \overline{v}$).[31] We restrict attention to cases in which sellers always find it optimal to serve all buyers who engage in search.[32]

4.1 Pricing decisions

When price discrimination is not possible, sellers choose their price quotes according to a mixed strategy that can no longer be conditioned on the buyer's valuation. Still, the expected prices paid by buyers depend on their search intensity, which might be different for high and low valuation consumers. We thus write $\rho(v)$ to denote the probability with which a buyer of valuation v searches once only. A seller's ex-ante expected profits from pricing at b are given by,

$$\begin{aligned}\pi(b) &= \alpha b\left[\frac{\rho(\overline{v})}{2} + (1-\rho(\overline{v}))(1-F(b))\right] G(c_0(\overline{v})) \\ &\quad + (1-\alpha) b\left[\frac{\rho(\underline{v})}{2} + (1-\rho(\underline{v}))(1-F(b))\right] G(c_0(\underline{v})).\end{aligned}$$

Profits are now weighted by the distribution of high and low valuation buyers in the market, conditional on search.

As shown in the following Proposition, there exists a symmetric equilibrium quote distribution which is analogous to the one in Proposition 1. The main difference is that ρ is now replaced by $\tilde{\rho}$, which reflects the weighted average of the different values of $\rho(v)$ across the population of buyers who search. In other words, firms price *as if* they were facing the "average" buyer. Additionally, the quote support is based on the low valuation consumer, instead of targetted to the consumer's own valuation.

Proposition 5 *Assume that sellers always find it optimal to serve all buyers who engage in search. With uniform prices, there is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by*

$$F(b) = 1 - \frac{1}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\frac{\underline{v}-b}{b},$$

with compact support $b \in \left[\frac{\underline{v}\tilde{\rho}}{2-\tilde{\rho}}, \underline{v}\right]$*, where*

$$\tilde{\rho} \equiv \beta\rho(\overline{v}) + (1-\beta)\rho(\underline{v}) \in (0,1)$$

is the weighted average $\rho(v)$ *in the market, with* β *representing the weight of the high valuation*

[31] It is easy to extend the results of the model to the case with a continuum of buyers with valuations v in a compact interval.

[32] In the alternative case, sellers would find it optimal not to serve the low valuation consumers with some positive probability, and the upper bound of the support would be $\overline{v}$. However, this would make low valuation consumers worse off under uniform prices as they would participate in search less often and would also face higher prices. We will arrive at the same conclusion without assuming that sellers prefer to leave some low types unserved.

consumers in the market,

$$\beta = \frac{\alpha G\left(c_0(\overline{v}, \tilde{\rho})\right)}{\alpha G\left(c_0(\overline{v}, \tilde{\rho})\right) + (1-\alpha)\, G\left(c_0(\underline{v}, \tilde{\rho})\right)}.$$

The above is an equilibrium if and only if

$$(1-\alpha)\, \underline{v}\rho\left(\underline{v}\right) G\left(c_0(\underline{v})\right) > \left(\overline{v} - \underline{v}\right) \alpha\rho\left(\overline{v}\right) G\left(c_0(\overline{v})\right).$$

Again, using the quote distribution characterized in Proposition 5, we can compute the expected price paid by a buyer with valuation v conditional on search.

Lemma 3 *Under uniform prices, the expected price (conditional on search) paid by the buyer with valuation $v \in \{\underline{v}, \overline{v}\}$ is given by*

$$E\left[p; v\right] = \underline{v}\tilde{\rho}\left[1 + \left(\rho\left(v\right) - \tilde{\rho}\right) X\left(\tilde{\rho}\right)\right]$$

where $X\left(\tilde{\rho}\right) \equiv \frac{-\ln(\tilde{\rho}/(2-\tilde{\rho}))}{2(1-\tilde{\rho})^2} - \frac{1}{1-\tilde{\rho}} > 0.$

The *competition effect* and the *probability effect* are now disentangled, as the former is captured by $\tilde{\rho}$ and the latter by $\rho\left(v\right)$. Sellers compete as if they faced an average buyer. Hence, the higher $\tilde{\rho}$, the higher the prices faced by all consumers. Even though high and low valuation buyers face the same prices conditional on search, their different willingness to search imply that they end up paying different prices. Indeed, buyers whose probability of searching once is above the average, $\rho\left(v\right) > \tilde{\rho}$, pay a higher price than the average buyer, and *viceversa.*

4.2 Search decisions

For a given pricing behavior of the sellers, buyers' search decisions also follow a cutoff strategy similar to the one in Proposition 2 above. However, since consumers now face uniform prices, the search thresholds have to be redefined, as stated next.

Proposition 6 *With uniform prices, the optimal search strategy of a buyer with valuation $v \in \{\underline{v}, \overline{v}\}$ follows a cut-off rule: for given $\tilde{\rho} \in (0,1)$, there exist $0 < c_1\left(v, \tilde{\rho}\right) < c_0\left(v, \tilde{\rho}\right) < v$ such that the buyer does not search if $c > c_0\left(v, \tilde{\rho}\right)$, she searches once if $c \in \left(c_1\left(v, \tilde{\rho}\right), c_0\left(v, \tilde{\rho}\right)\right]$ or she searches twice otherwise. Furthermore, $c_1\left(\underline{v}, \tilde{\rho}\right) = c_1\left(\overline{v}, \tilde{\rho}\right)$ and $c_0\left(\overline{v}, \tilde{\rho}\right) = c_0\left(\underline{v}, \tilde{\rho}\right) + \left(\overline{v} - \underline{v}\right)$.*

Banning price discrimination has a two-fold effect on the search strategy. First, through the effect on sellers' pricing behavior, the critical thresholds for the search strategy are now a function of the average $\tilde{\rho}$ rather than being buyer-specific. Second, since all buyers face equal prices, the thresholds for engaging in search, c_0, only differ across buyers in how much they value the good. Last, the threshold for searching twice conditional on search, c_1, is the same for all buyers given that they enjoy the good regardless of whether they search once or twice. Hence, we can simply write $c_1\left(\tilde{\rho}\right)$.

4.3 Equilibrium characterization

The equilibrium values of $\rho^*(v)$ are now found as the solution to the following system of equations, one for the high, one for the low, and one for the average valuation buyer in the market:

$$\rho^*(\overline{v}) = 1 - \frac{G(c_1(\tilde{\rho}^*))}{G(c_0(\overline{v}, \tilde{\rho}^*))} \tag{3}$$

$$\rho^*(\underline{v}) = 1 - \frac{G(c_1(\tilde{\rho}^*))}{G(c_0(\underline{v}, \tilde{\rho}^*))} \tag{4}$$

$$\tilde{\rho}^* = 1 - \frac{G(c_1(\tilde{\rho}^*))}{\alpha G(c_0(\overline{v}, \tilde{\rho}^*)) + (1-\alpha) G(c_0(\underline{v}, \tilde{\rho}^*))} \tag{5}$$

Analogously to Proposition 3, the next proposition shows that there exists a SPNE in the game with uniform prices.

Proposition 7 *With uniform prices, there exists a symmetric SPNE in which sellers price as stated in Proposition 5 and buyers search as stated in Proposition 2. Conditional on participating in search, the probability that the buyer asks for one quote is given by the solution to the system of equations (3) to (5). A sufficient condition for the equilibrium to be unique is that the elasticity of the search cost distribution G, $\varepsilon(c) \equiv cg(c)/G(c)$, is increasing in c.*

Equations (3) to (5) allow us to conclude that, in equilibrium, conditional on search, buyers with a high valuation search less than buyers with low valuation. Together with Lemma 3, this implies that the former pay higher expected prices once they search. Indeed, the difference in the prices paid by high and low valuation consumers is a function of the difference between their search intensities; it is also proportional to the the average consumer's search threshold c_1. This is formally stated next.

Lemma 4 *(i) In equilibrium, the high valuation consumers search less than the low valuation consumers, $\rho^*(\overline{v}) > \tilde{\rho}^* > \rho^*(\underline{v})$. (ii) Hence, conditional on search, the high valuation buyers pay higher expected prices than the low valuation consumers, $E[p;\overline{v}] - E[p;\underline{v}] = [\rho^*(\overline{v}) - \rho^*(\underline{v})]\, c_1(\tilde{\rho}^*) > 0$. (iii) An increase in the mass of the high types reduces the search intensity of the average consumer: $\tilde{\rho}^*$ is increasing in α.*

Sellers would like to compete more (less) fiercely for consumers with a low (high) valuation knowing that, conditional on search, they are more (less) likely to have searched twice. However, the ban on price discrimination stops them from doing so. Indeed, sellers price as if all consumers searched like the average consumer, implying that high (low) valuation consumers pay lower (higher) prices than if sellers priced according to their actual search behavior. Yet, as compared to low valuation buyers, high valuation buyers pay higher prices conditional on search as they find it optimal to search less. An increase in the mass of high types reduces the search intensity of the average buyer, and hence leads to an overall price increase.

All the above has implications for the comparison of price discrimination versus uniform prices, an issue to which we turn next.

5 Price Discrimination *versus* Uniform Prices

We are now ready to understand the effects of banning/allowing for price discrimination by comparing the results obtained in Sections 3 and 4. For the sake of exposition, we add the subscripts d and u to distinguish the cases with discriminatory or uniform prices, respectively.

For the sake of clarity, we focus attention on the case in which the elasticity of the search cost distribution is everywhere increasing (e.g., search costs are uniformly distributed, and there is a positive mass of shoppers). Also, recall that for the analysis of uniform prices, we are focusing on the case in which all consumers are served conditional on search.

We start by looking at the effects on the low valuation consumers.

Proposition 8 *Suppose that the elasticity of the search cost distribution is increasing in c. In equilibrium,* $\widetilde{\rho}^u \geq \rho^d(\underline{v}) \geq \rho^u(\underline{v})$. *It follows that* $G(c_0^u(\underline{v})) < G\left(c_0^d(\underline{v})\right)$.

With uniform prices, consumers with low valuation are made worse off from being pooled with high valuation consumers. Conditional on search, low valuation consumers tend to search more under uniform pricing (as reflected in a higher probability of searching twice, $1-\rho^u(\underline{v}) \geq 1-\rho^d(\underline{v})$), and they also tend to pay higher prices (as reflected in $\widetilde{\rho}^u \geq \rho^d(\underline{v})$). This has a translation on how often they engage in search. As formally stated in the Proposition, buyers with low valuation are less likely to engage in search with uniform prices than under price discrimination, $G(c_0^u(\underline{v})) < G\left(c_0^d(\underline{v})\right)$.

Figure 3 illustrates these results for various values of α (only those for which the necessary and sufficient condition for equilibrium is satisfied; see Proposition 5). Regarding search intensity (upper panel), the low valuation consumers (dashed lines) always search less intensively under discrimination (grey) than under uniform prices (black). Regarding search participation (lower panel), the low valuation consumers (dashed lines) always engage in search more often under discrimination (grey) than under uniform prices (black).

Let us now turn attention to the high valuation consumers. Conditional on search, they tend to search less intensively with uniform prices than under price discrimination (as reflected in a lower probability of searching twice, $1-\rho^u(\overline{v}) \leq 1-\rho^d(\overline{v})$). However, whether they pay higher or lower prices depends on parameter values. In particular, if the mass α of high types is sufficiently small (high), the high types pay lower (higher) prices with uniform prices than under price discrimination. Intuitively, under uniform prices, the high valuation consumers benefit substantially from being pooled with the low valuation consumers, both because the low types search more intensively and because the price distribution is now capped at the low types' maximum willingness to pay, $\underline{v}$. The lower prices they face induce the high types to participate in search more often.

However, there are also cases in which the high types' willingness to search is lower under uniform prices. In spite of being pooled with consumers who search more intensely than in the discriminatory case, their own search behavior might dilute the search intensity in the average effect, leading to cases with $\rho^d(\overline{v}) \leq \widetilde{\rho}^u \leq \rho^u(\overline{v})$. This is more likely as the share of high valuation consumers (α) increases, as it implies that $\widetilde{\rho}^u$ approaches $\rho^u(\overline{v})$. On the contrary, as α goes to zero,

$\widetilde{\rho}^u$ approaches $\rho^u(\underline{v})$ and it is thus below $\rho^d(\overline{v})$. Figure 3 also illustrates these results, with the high valuation consumers depicted through dotted lines.

Putting the above results together, it unambiguously follows that, when the elasticity of the search cost distribution is increasing, a ban on price discrimination makes low valuation consumers worse off. In contrast, the high types tend to be better off, at least when there are not too many of them. Given these countervailing effects across consumers, it is not possible in general to provide a welfare ranking. For this reason, we resort to numerical solutions to illustrate the potential welfare effects of a ban on price discrimination. As shown in Figure 4 (upper panel), a ban on price discrimination allows the high valuation consumers pay lower prices, at the expense of the low valuation consumers; as shown in the lower panel, overall welfare goes up because the reduction in the surplus of the low valuation consumers is more than compensated by the increase in the surplus of the high valuation consumers.

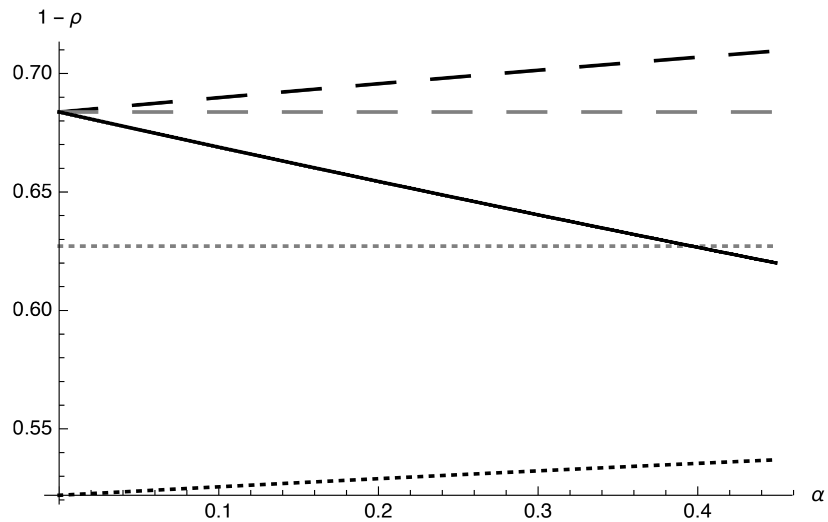
6 Conclusions

In this paper we have built a model of search with third-degree price discrimination that sheds light on the determinants of price differences across buyers, an issue that has become increasingly relevant for competition policy.

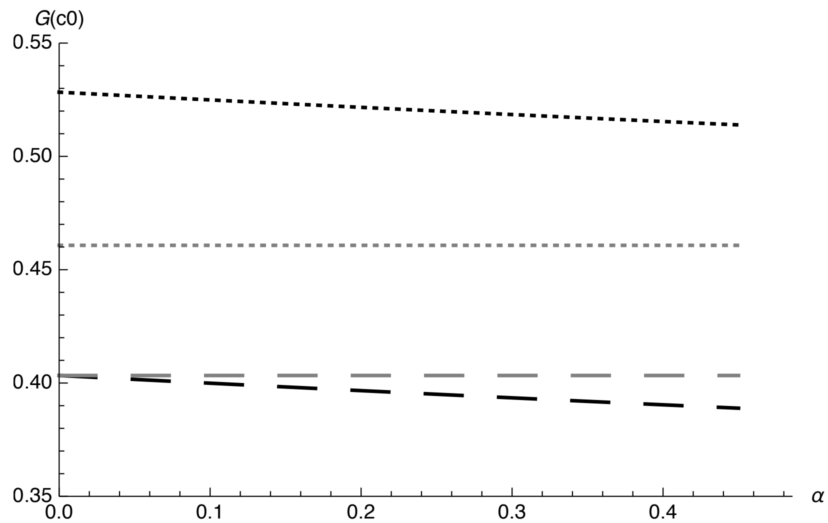
In our model, differences in buyers' willingness to pay introduce heterogeneity in buyers' willingness to search, and this opens up the scope for competition-related price discrimination. In particular, sellers compete more fiercely for those buyers with a higher willingness to search. The key to assessing the effects of price discrimination on the various consumers is thus to understand whether buyers with higher valuations have stronger or weaker willingness to search (conditional on search) as compared to buyers with lower valuations. On the one hand, for a given search cost, high valuation consumers are expected to search more as the gains from search are proportional to their valuations. On the other, low valuation consumers participate in search only if their search costs are sufficiently low, in which case they are also expected to search more. In this paper we have shown that the dominant of these two forces ultimately depends on the shape of the search cost distribution function.

In particular, we have identified a simple condition to predict the relationship between the intensity of competition and buyers' valuations: if the elasticity of the search cost distribution is decreasing (increasing) over the relevant range, then firms compete more (less) fiercely for buyers with higher valuation. If search costs are normally distributed, the elasticities depict an inverted-U shape. Hence, prices conditional on search are also non-monotonic in the buyers' valuations, with consumers in the middle range facing less competition than either those buyers with very low or very high valuations. Intuitively, buyers with average valuations do not benefit as much as the ones with low valuations from signaling low search costs when they engage in search, nor do they benefit as much as the ones with high valuations from signaling higher gains from search. The effects of buyers' willingness to buy on the intensity of competition among firms also determine the

Figure 3: Search (intensive and extensive margin): uniform vs. discriminatory

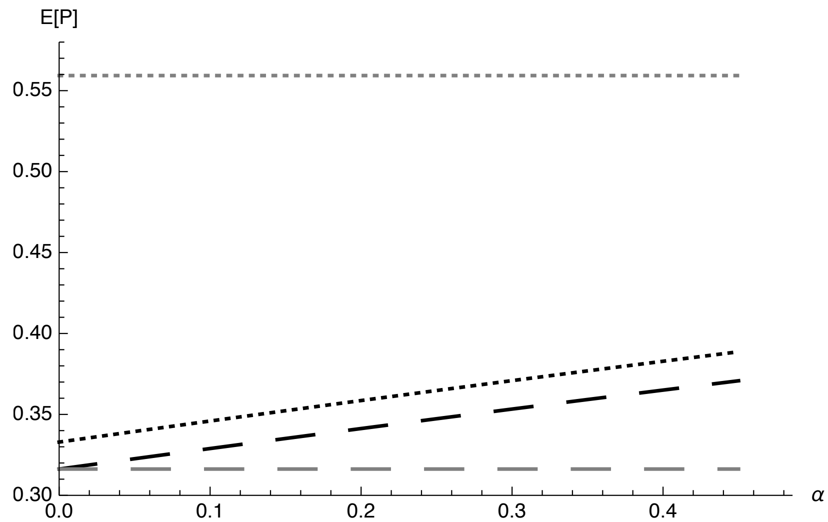


(a) Search: $1 - \rho$ for different cases. Solid black line shows $1 - \widetilde{\rho}^u$ (weighted average), dotted lines show equilibrium for high types (uniform in black, discriminatory in grey). High-value consumers search substantially less under uniform pricing due to more favorable prices. Dashed lines show search intensity for low valuation consumers, which increases under uniform pricing. Discriminatory probabilities in grey are independent of α.

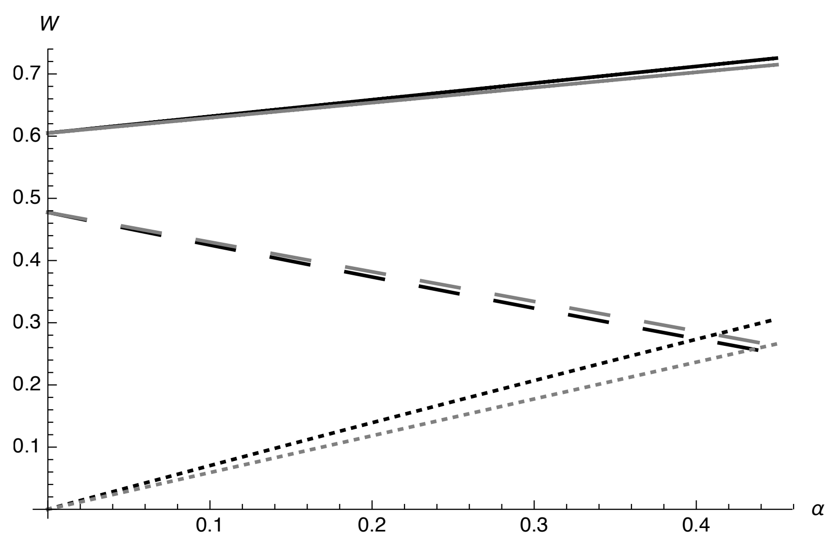


(b) Mass of consumers who search at least once $G(c_0)$, under different values of α. High-value consumers (dotted lines) participate more in the market under uniform pricing (black). Low-value consumers (dashed lines) participate less under uniform pricing (black) but the effect is smaller. Gross consumer surplus increases.

Figure 4: Expected prices and welfare: uniform vs. discriminatory



(a) Prices: comparison between expected prices under uniform pricing (black) vs. discriminatory pricing (grey). High valuation consumers subtantially benefit from uniform prices (dotted black line) compared to discriminatory pricing (dotted grey line), whereas low valuation consumers lose from it as soon as some high valuation consumers enter the market, due to their lower search intensity.



(b) Welfare comparison: the solid lines show welfare under discrimination (grey) and under uniform prices (black). High valuation consumers' surplus increases with uniform prices (dotted black line) compared to discriminatory pricing (dotted grey line), whereas low valuation consumers lose surplus under uniform prices. Welfare and surplus levels are weighted by α and $1-\alpha$ to represent total welfare and surplus measures.

likelihood of search in the first place, and thus the distribution of the valuations of those buyers who endogenously decide to engage in search.

Our analysis provides predictions regarding the effects of price discrimination on the various consumers. We have found that low valuation consumers tend to be worse off under uniform prices both because they face higher prices and because they are induced to search more. The contrary applies to the high valuation buyers. We have provided a parametrization (uniformly distributed search costs with shoppers) under which a ban on price discrimination would be welfare improving, as the increase in the surplus of the high valuation buyers' dominates. However, since these predictions rely on the elasticities of the search cost distribution, which need not coincide across all markets, it is simply not possible to provide a single answer to the general question of whether and how a ban on price discrimination would affect overall welfare. Inexorably, the answer has to rely on industry specific studies that shed light on consumers' search behavior as well as on the distribution of the various consumers' types in the market.

References

Armstrong, M. and Vickers, J. (2018). Discriminating against captive customers. *mimeo, Oxford University.*

Armstrong, M. and Zhou, J. (2016). Search deterrence. *Review of Economic Studies*, 83(1):26–57.

Burdett, K. and Judd, K. L. (1983). Equilibrium price dispersion. *Econometrica*, 51(4):pp. 955–969.

De los Santos, B., Hortaçsu, A., and Wildenbeest, M. R. (2012). Testing models of consumer search using data on web browsing and purchasing behavior. *American Economic Review*, 102(6):2955–2980.

Diamond, P. (1971). A model of price adjustment. *Journal of Economic Theory*, 3(2):156–168.

European Commission (2016). Geo-blocking practices in e-commerce. *Commission Staff Working Document*, SWD(2016) 70 final.

Fabra, N. and Montero, J. P. (2019). Product choice, price discrimination and market frictions. *CEPR Discussion Paper.*

Gugler, K., Heim, S., Jansen, M., and Liebensteiner, M. (2019). Incumbency advantages: Price dispersion, price discrimination and consumer search. *mimeo, Vienna University.*

Hong, H. and Shum, M. (2006). Using price distributions to estimate search costs. *The RAND Journal of Economics*, 37(2):257–275.

Hortaçsu, A. and Syverson, C. (2004). Product differentiation, search costs, and competition in the mutual fund industry: A case study of s&p 500 index funds. *Quarterly Journal of Economics*, 119(2):403–456.

Janssen, M. and Rasmusen, E. (2002). Bertrand competition under uncertainty. *Journal of Industrial Economics*, 50(1):11–21.

Janssen, M. and Rushidi, E. (2018). Wholesale price discrimination and consumer search. *mimeo, University of Vienna*.

Janssen, M. C. and Moraga, J. L. (2004). Strategic pricing, consumer search and the number of firms. *The Review of Economic Studies*, 71(4):1089–1118.

Lauermann, S. and Wolinsky, A. (2016). Search with adverse selection. *Econometrica*, 84(1):243315.

Lauermann, S. and Wolinsky, A. (2017). Bidder solicitation, adverse selection and the failure of competition. *The American Economic Review*, 107(6):13991429.

Moraga, J. L., Sandor, Z., and Wildenbeest, M. (2017a). Nonsequential search equilibrium with search cost heterogeneity. *International Journal of Industrial Organization*, 50:392414.

Moraga, J. L., Sandor, Z., and Wildenbeest, M. (2017b). Prices and heterogeneous search costs. *RAND Journal of Economics*, 48(1):125146.

Moraga, J. L. and Wildenbeest, M. R. (2008). Maximum likelihood estimation of search costs. *European Economic Review*, 52:820–48.

Petrikaite, V. (2018). Consumer obfuscation by a multiproduct firm. *RAND Journal of Economics*, 49:206–223.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies*, 68(1):205–229.

Sorensen, A. T. (2000). Equilibrium price dispersion in retail markets for prescription drugs. *Journal of Political Economy*, 108(4):833–850.

Stahl, D. (1989). Oligopolistic pricing with sequential consumer search. *American Economic Review*, 79(4):700–712.

Varian, H. R. (1980). A model of sales. *American Economic Review*, 70(4):651–59.

A Appendix: Proofs

Proof of Lemma 1. Suppose $\rho = 1$. Then, the seller knows that it is a monopolist and hence charges the reservation price. However, it would leave no surplus for the buyer, so her participation constraint would not be satisfied. Hence, $\rho = 0$ cannot be part of an equilibrium in which the buyer has decided to participate. Suppose $\rho = 0$. Then, there is Bertrand competition with both sellers quoting prices equal to marginal cost. However, knowing that all sellers choose the same quote, the buyer would have incentives to deviate and search less in order to save on search costs. Hence, $\rho = 0$ cannot be part of an equilibrium. As a consequence, in equilibrium, we must have $\rho \in (0, 1)$.

■

Proof of Proposition 1. Let sellers choose the distribution of quotes $F(b)$ over the interval $\left[\underline{b}, \overline{b}\right]$. Standard arguments imply that this distribution must be atomless. In particular, if sellers put mass on a given quote, there would be a positive probability of a tie. If such a quote is above marginal costs, each seller would be better off by slightly reducing its quote below that level: this would have a minor effect on its profits if the buyer has asked for one quote only, but would guarantee that the seller makes the sale whenever the buyer has asked for two quotes. Putting mass at marginal cost cannot be part of an equilibrium either, given that each seller would be able to make the sale at a higher price whenever the buyer has asked for one quote only.

Seller i's profits from quoting a price b when rivals are choosing quotes according to $F(b)$ are given by:[33]

$$\pi(b) = b\left(\frac{\rho}{2} + (1-\rho)(1 - F(b))\right).$$

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi(b; q) = \overline{\pi} \text{ for all } b \in \left[\underline{b}, \overline{b}\right].$$

Furthermore, as $\pi\left(\overline{b}\right) = \overline{b}\frac{\rho}{2}$ is increasing in $\overline{b}$, it follows that $\overline{b} = v$. Hence, $\overline{\pi} = \frac{v\rho}{2}$. From $\pi(\underline{b}) = \underline{b}\left(\frac{\rho}{2} + (1-\rho)\right) = \overline{\pi} = \frac{v\rho}{2}$ it also follows that $\underline{b} = \frac{v\rho}{2-\rho}$. Accordingly, the support of the equilibrium mixed strategy is $b \in \left[\frac{v\rho}{2-\rho}, v\right]$.

To obtain the equilibrium quote distribution, from $\pi(b) = \frac{v\rho}{2}$, it follows that

$$F(b) = 1 - \frac{1}{2}\frac{\rho}{1-\rho}\frac{v-b}{b},$$

with density

$$f(b) = \frac{1}{2}\frac{\rho}{1-\rho}\frac{v}{b^2}.$$

For $\rho \to 1$, the equilibrium collapses to the reservation price, whereas for $\rho \to 0$, the equilibrium collapses to marginal costs. Sellers' expected profits when quoting a price to a buyer of valuation v are thus $v\rho$. ■

Proof of Lemma 2. We first derive the distribution of the price paid by the buyer (conditional on search), which we denote as F_p, using the quote distribution characterized above. When the buyer has searched n times, the distribution is $(1 - (1 - F(b))^n)$. Hence, given that buyers only

[33] Results would be the same if we instead computed the firm's expected profits using probabilities conditional on having received a quote request. Let $x = \frac{\rho/2}{1-\rho/2}$ be the conditional probability of receiving a quote request from a buyer who has only asked for one quote only. Expected profits would thus be written as $\pi(b) = bq(x + (1-x)(1 - F(b)))$. The equilibrium price distribution would be $F(b) = 1 - \frac{x}{1-x}\frac{1-b}{b}$, with b in the compact support $p \in [x, 1]$. While both formulations are equivalent, we believe that using the unconditional probability ρ allows for a more intuitive interpretation of the results.

get one quote with probability ρ, one finds the distribution the expected price as,

$$
\begin{aligned}
F_p(b) &= \rho F(b) + (1-\rho)\left(1-(1-F(b))^2\right) \\
&= \frac{(b(\rho-2)+v\rho)(b(\rho-2)-v\rho)}{(2b)^2(1-\rho)},
\end{aligned}
$$

$$
f_p(b) = \frac{v^2\rho^2}{2b^3(1-\rho)}.
$$

Note that the price distribution $F_p(b)$ is decreasing in ρ,

$$
\frac{\partial F_p(b)}{\partial \rho} = -\frac{\rho(2-\rho)(v-b)(v+b)}{4b^2(1-\rho)^2} < 0.
$$

With this expression we can compute the expected price paid by the buyer (conditional on search) as

$$
\int_{\underline{b}}^{\overline{b}} b f_p(b)\, db = v\rho.
$$

■

Proofs of Proposition 2. A buyer who searches once or twice derives utility

$$
\begin{aligned}
u_1 &= \left(v - \int_{\underline{b}}^{\overline{b}} b dF(b)\right) q - c \\
u_2 &= \left(v - \int_{\underline{b}}^{\overline{b}} 2b(1-F(b))\, dF(b)\right) q - 2c,
\end{aligned}
$$

with $F(b)$ and $\left[\underline{p}, \overline{b}\right]$ as characterized in Proposition 1. The functions $u_1(c)$ and $u_2(c)$ are linear in c, with constant slope -1 and -2, respectively, and $u_2(0) > u_1(0) > 0$. It follows that there exist $0 < c_1 < c_0 < v$ such that (i) $0 > u_1(c) > u_2(c)$ for $c > c_0$; (ii) $u_1(c) \geq \max\{u_2(c), 0\}$ for $c \in (c_1, c_0]$ and (iii) $u_2(c) \geq u_1(c) > 0$ for $c \leq c_1$. More specifically, c_0 and c_1 are implicitly defined by $u_1(c_0) = 0$ and $u_1(c_1) = u_2(c_1)$. We next derive explicit formula, using the equilibrium quote distribution characterized in Proposition 1.

Conditional on observing one quote, the expected utility is

$$
\begin{aligned}
u_1 &= \left(v - \int_{\underline{b}}^{\overline{b}} b dF(b)\right) - c \\
&= v\left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right) - c.
\end{aligned}
$$

Hence, to be indifferent between searching or not:

$$\begin{aligned} c_0 &= v - \frac{1}{2}\frac{v\rho}{1-\rho}\int_{v\frac{\rho}{2-\rho}}^{v}\frac{1}{b}db \\ &= v\left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right). \end{aligned}$$

The threshold c_0 is decreasing in ρ, with $\lim_{\rho\to 0} c_0 = v$ and $\lim_{\rho\to 1} c_0 = 0$. For future reference,

$$\lim_{\rho\to 1}\frac{\partial c_0}{\partial\rho} = -v.$$

Conditional on observing two quotes, the expected price is the minimum of the two, so the utility is

$$\begin{aligned} u_2 &= \left(v - \int_{\underline{b}}^{\bar{b}} 2b\left(1-F\left(b\right)\right)dF\left(b\right)\right) - 2c \\ &= v - \frac{v\rho^2}{2\left(1-\rho\right)^2}\left(\frac{2-2\rho}{\rho} + \ln\frac{\rho}{2-\rho}\right) - 2c. \end{aligned}$$

Equating the two utilities and solving for c:

$$c_1 = v\frac{\rho}{1-\rho}\left(\frac{1}{2}\frac{1}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right) - 1\right) = v - \frac{c_0\left(v\right)}{1-\rho}$$

Taking the derivative with respect to ρ,

$$\frac{\partial c_1}{\partial\rho} = -\frac{1}{\left(1-\rho\right)^2}\left(c_0\left(v\right) + \left(1-\rho\right)\frac{\partial c_0\left(v\right)}{\partial\rho}\right)$$

shows that c_1 increases in ρ up to $\rho = 0.365v$ and decreases thereafter, with $\lim_{\rho\to 0} c_1 = \lim_{\rho\to 1} c_1 = 0$. For future reference,

$$\lim_{\rho\to 1}\frac{\partial c_1}{\partial\rho} = -\frac{v}{3}\cdot$$

Taking the difference between the two thresholds:

$$\begin{aligned} c_0 - c_1 &= c_0\frac{2-\rho}{1-\rho} - v \\ &= v\left(\frac{2-\rho}{1-\rho}\left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right) - 1\right) > 0\cdot \end{aligned}$$

It follows that

$$\frac{\partial c_0}{\partial v} - \frac{\partial c_1}{\partial v} > 0$$

The difference is decreasing in ρ, with $\lim_{\rho\to 0}\left(c_0 - c_1\right) = v$ and $\lim_{\rho\to 1}\left(c_0 - c_1\right) = 0$. For future reference,

$$\frac{\partial c_0}{\partial \rho}-\frac{\partial c_1}{\partial \rho}=v\frac{2-2\rho-\ln\left(\frac{2-\rho}{\rho}\right)}{(1-\rho)^3}<0. \tag{6}$$

Also for future reference, let us note that the following inequality is always satisfied

$$\frac{c_1}{c_0}>\frac{\frac{\partial c_1}{\partial \rho}}{\frac{\partial c_0}{\partial \rho}}. \tag{7}$$

■

Proof of Proposition 3. We need to show that there exists a solution to (1) in $(0,1)$. To show that condition (1) has a solution, note that $\lim_{\rho\to 0} c_1=0$ and $\lim_{\rho\to 0} c_0=v$. Hence, when $\underline{c}<0$

$$\lim_{\rho\to 0}\left(1-\frac{G(c_1)}{G(c_0)}\right)=1-\frac{G(0)}{G(v)}\in(0,1),$$

whereas when $\underline{c}=0$,

$$\lim_{\rho\to 0}\left(1-\frac{G(c_1)}{G(c_0)}\right)=1.$$

Furthermore, $\lim_{\rho\to 1} c_1=\lim_{\rho\to 1} c_0=0$, implying that $G(c_1)=G(c_0)$. Hence, when $\underline{c}<0$

$$\lim_{\rho\to 1}\left(1-\frac{G(c_1)}{G(c_0)}\right)=0.$$

When $\underline{c}=0$, both $G(c_1)$ and $G(c_0)$ take the value 0 for $\rho\to 1$, so $\lim_{\rho\to 1}\frac{G(c_1)}{G(c_0)}$ is undefined. Applying l'Hôpital,

$$\begin{aligned}\lim_{\rho\to 1}\left(1-\frac{G(c_1)}{G(c_0)}\right) &= 1-\lim_{\rho\to 1}\frac{g(c_1)\frac{\partial c_1}{\partial \rho}}{g(c_0)\frac{\partial c_0}{\partial \rho}}\\ &= 1-\frac{g(0)}{g(0)}\frac{v/3}{v}\\ &= \frac{2}{3}<1.\end{aligned}$$

Under regularity conditions on G that ensure continuity, note that: (i) for $\underline{c}=0$, the function takes the value 1 for $\rho\to 0$ and a lower value for $\rho\to 1$, and (ii) for $\underline{c}<0$, the function takes a strictly positive value for $\rho\to 0$ and zero for $\rho\to 1$. Hence, in both cases, the function must cross the 45 degree line at some $\rho^*\in(0,1)$. Hence, there exists an interior solution to (1).

[Uniqueness] A sufficient condition for equilibrium uniqueness is that G is log-convex as in this case the RHS of equation (1) is everywhere decreasing in ρ. Hence, it crosses the 45-degree line only once. In detail, the general derivative for the function is

$$\frac{\partial\left(1-\frac{G(c_1)}{G(c_0)}\right)}{\partial \rho}=\frac{g(c_0)G(c_1)\frac{\partial c_0}{\partial \rho}-g(c_1)G(c_0)\frac{\partial c_1}{\partial \rho}}{(G(c_0))^2}.$$

Since the denominator is always positive, we focus on the numerator. It will be negative as long as,

$$\frac{g(c_0)}{G(c_0)}\frac{\partial c_0}{\partial \rho} < \frac{g(c_1)}{G(c_1)}\frac{\partial c_1}{\partial \rho}. \tag{8}$$

Using our previous results,

$$\frac{\partial c_0}{\partial \rho} < 0 \text{ and } \frac{\partial c_0}{\partial \rho} < \frac{\partial c_1}{\partial \rho}.$$

Re-writing equation (8), the RHS of equation (1) is everywhere decreasing in ρ iff

$$\frac{\frac{g(c_0)}{G(c_0)}}{\frac{g(c_1)}{G(c_1)}} > \frac{\frac{\partial c_1}{\partial \rho}}{\frac{\partial c_0}{\partial \rho}}.$$

A sufficient condition for this to be satisfied is that the elasticity of G is increasing. In particular, if the elasticity is increasing,

$$\frac{g(c_0)}{G(c_0)}c_0 > \frac{g(c_1)}{G(c_1)}c_1 \Leftrightarrow \frac{\frac{g(c_0)}{G(c_0)}}{\frac{g(c_1)}{G(c_1)}} > \frac{c_1}{c_0} > \frac{\frac{\partial c_1}{\partial \rho}}{\frac{\partial c_0}{\partial \rho}},$$

where the last inequality follows from equation (7).

If the elasticity of G is decreasing, the right hand side of equation (8) is decreasing in ρ up to $\rho \leq 0.365v$ as in this case $\partial c_1/\partial \rho > 0$. However, for $\rho > 0.365v$ the right hand side of equation (1) can eventually become positively sloped. We cannot then resort to the same argument to show uniqueness.

A less stringent sufficient condition for uniqueness is that the slope of the right hand side of (1) is below 1. This condition becomes

$$\frac{G(c_0)}{G(c_1)} > \frac{g(c_0)}{G(c_0)}\frac{\partial c_0}{\partial \rho} - \frac{g(c_1)}{G(c_1)}\frac{\partial c_1}{\partial \rho}.$$

If condition (8) is satisfied, the RHS of the above equation is negative so that the condition is always satisfied. If the RHS is positive (meaning that the schedule $1 - G(c_1)/G(c_0)$ is positively sloped), the condition essentially requires that G is not too concave. ■

Proof of Proposition 4. For ρ^* to be increasing (decreasing) in v, the RHS of the equilibrium condition (1) must be increasing (decreasing) in v. This is the case if and only if

$$\frac{\partial\left(1 - \frac{G(c_1)}{G(c_0)}\right)}{\partial v} = -\frac{c_1 G(c_0) g(c_1) - c_0 G(c_1) g(c_0)}{v(G(c_0))^2} > 0.$$

where we have used the fact that both c_1 and c_0 are linear in v so that their derivatives are simply c_1/v and c_0/v respectively. Since the denominator is positive, we focus attention on the numerator, which can be re-written as

$$G(c_0)\, G(c_1) \left[c_0 \frac{g(c_0)}{G(c_0)} - c_1 \frac{g(c_1)}{G(c_1)} \right] > 0,$$

or using the expression for $\varepsilon(c) \equiv c\frac{g(c)}{G(c)}$,

$$G(c_0)\, G(c_1) \left[\varepsilon(c_0) - \varepsilon(c_1)\right] > 0.$$

It follows that a sufficient condition for ρ^* to be increasing (decreasing) in v is that the term in brackets, evaluated at ρ^*, is positive (negative), i.e., $\varepsilon(c_0) > \varepsilon(c_1)$ ($<$). Hence, as v goes up, the schedule crosses the 45-degree line at a higher (smaller) value of ρ. If the elasticity $\varepsilon(c)$ is constant, changes in v do not move the schedule, and hence the equilibrium remains unchanged. Note that if G is log-convex, then $\varepsilon(c_0) > \varepsilon(c_1)$ so that ρ^* is increasing in v. ■

Proof of Proposition 5. Similar arguments are those in Proposition 1 allow us to conclude that there does not exist an equilibrium in pure strategies.

First suppose that sellers never find it optimal to charge prices above $\underline{v}$ (later we will make it explicit the condition for this to be the case). Let seller i's profits from quoting a price $b \in \left[\underline{b}, \overline{b}\right]$ when the rival is choosing quotes according to $F(b)$ be given by:

$$\begin{aligned} \pi(b) &= \alpha b \left[\frac{\rho(\overline{v})}{2} + (1-\rho(\overline{v}))(1-F(b)) \right] G(c_0(\overline{v})) \\ &\quad + (1-\alpha)\, b \left[\frac{\rho(\underline{v})}{2} + (1-\rho(\underline{v}))(1-F(b)) \right] G(c_0(\underline{v})) \end{aligned}$$

Letting

$$\begin{aligned} x &= \alpha\rho(\overline{v})\, G(c_0(\overline{v})) + (1-\alpha)\,\rho(\underline{v})\, G(c_0(\underline{v})) \\ y &= \alpha G(c_0(\overline{v})) + (1-\alpha)\, G(c_0(\underline{v})) \end{aligned}$$

Profits can be re-written as

$$\pi(b) = b\left(x/2 + (y-x)(1-F(b))\right).$$

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi(b) = \overline{\pi} \text{ for all } b \in \left[\underline{b}, \overline{b}\right].$$

Furthermore, as $\pi\left(\overline{b}\right) = \overline{b}x/2$ is increasing in $\overline{b}$, it follows that $\overline{b} = \underline{v}$. Hence, $\overline{\pi} = \underline{v}x/2$. From $\pi\left(\underline{b}\right) = \underline{b}\left(y - x/2\right) = \overline{\pi} = \underline{v}x/2$ it also follows that $\underline{b} = \frac{\underline{v}\frac{x}{y}}{2-\frac{x}{y}}$. To obtain the equilibrium quote distribution, from $\pi(b) = \underline{v}x/2$, it follows that

$$F(b) = 1 - \frac{1}{2}\frac{x}{y-x}\frac{\underline{v}-b}{b}.$$

Renormalize it by defining

$$\begin{aligned}\tilde{\rho} &\equiv \frac{x}{y} = \frac{\alpha\rho(\overline{v})\,G(c_0(\overline{v})) + (1-\alpha)\,\rho(\underline{v})\,G(c_0(\underline{v}))}{\alpha G(c_0(\overline{v})) + (1-\alpha)\,G(c_0(\underline{v}))} \\ &= \frac{\alpha G(c_0(\overline{v}))}{\alpha G(c_0(\overline{v})) + (1-\alpha)\,G(c_0(\underline{v}))}\rho(\overline{v}) + \frac{(1-\alpha)\,G(c_0(\underline{v}))}{\alpha G(c_0(\overline{v})) + (1-\alpha)\,G(c_0(\underline{v}))}\rho(\underline{v})\end{aligned} \tag{9}$$

(i.e., $\tilde{\rho}$ is the weighted average $\rho(\overline{v})$ and $\rho(\underline{v})$) so that

$$F(b) = 1 - \frac{1}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\frac{\underline{v}-b}{b}.$$

Suppose instead one of the sellers deviates and charges a price above $\underline{v}$. Since its profits would be increasing in its price, the firm would set $\overline{v}$. In this case, its expected profits would be

$$\alpha\overline{v}\frac{\rho(\overline{v})}{2}G(c_0(\overline{v}))$$

as the firm would not serve the low valuation consumers. Therefore, to rule out this deviation, we require the gain over the low valuation consumers to be greater than the loss out of the high valuation consumers,

$$(1-\alpha)\,\underline{v}\rho(\underline{v})\,G(c_0(\underline{v})) > (\overline{v}-\underline{v})\,\alpha\rho(\overline{v})\,G(c_0(\overline{v}))$$

The above condition has to be satisfied for the above to constitute an equilibrium. Note that it is more easily satisfied the lower α. ■

Proof of Lemma 3.

We follow the same steps as in the discrimination case. We first derive the distribution of the price paid by the buyer with valuation v (conditional on search), which we denote as $F_p(v)$, using the quote distribution characterized in the case of no discrimination. When the buyer has searched n times, the distribution is $(1-(1-F(b))^n)$. Hence, given that buyers with valuation v only get one quote with probability $\rho(v)$ but sellers price as if they faced a buyer with $\tilde{\rho}$, one finds the distribution the expected price as,

$$\begin{aligned}F_p(b;v) &= \rho(v)\,F(b) + (1-\rho(v))\left(1-(1-F(b))^2\right) \\ &= (\underline{v}\tilde{\rho} - b(1-\tilde{\rho}))\,\frac{(b-\underline{v})(1-\rho(v))\,\tilde{\rho} - 2b(1-\tilde{\rho})}{(2b)^2(1-\tilde{\rho})^2}.\end{aligned}$$

With density,

$$f_p(b) = \underline{v}\tilde{\rho}\frac{\underline{v}\rho(v)(1-\rho(v)) - b(\tilde{\rho}-\rho(v))}{2b^3(1-\tilde{\rho})^2}.$$

With this expression we can compute the expected price paid by the buyer with valuation v (conditional on search) as,

$$\begin{aligned} E[p;v] &= \int_{\underline{b}}^{\bar{b}} b f_p(b;v)\,db \\ &= \underline{v}\tilde{\rho}\left(1 + (\rho(v)-\tilde{\rho})\left(-\frac{1}{1-\tilde{\rho}} - \frac{1}{2(1-\tilde{\rho})^2}\ln\left(\frac{\tilde{\rho}}{2-\tilde{\rho}}\right)\right)\right). \end{aligned}$$

Clearly, for the average consumer, $\rho(v) = \tilde{\rho}$, we obtain a similar expression as in the discrimination case, with an expected price conditional on search equal to $\underline{v}\tilde{\rho}$ (the only difference being that the relevant valuation is now $\underline{v}$).

The difference in the expected prices paid (conditional on search) by the high and low valuation consumers is given by:

$$E[p;\overline{v}] - E[p;\underline{v}] = (\rho(\overline{v}) - \rho(\underline{v}))\,\underline{v}\tilde{\rho}\left(-\frac{1}{1-\tilde{\rho}} - \frac{1}{2(1-\tilde{\rho})^2}\ln\left(\frac{\tilde{\rho}}{2-\tilde{\rho}}\right)\right) > 0.$$

■

Proof of Proposition 6. We follow the same steps as in the discrimination case. The utilities from searching once or twice for a customer with valuation v are now given by:

$$\begin{aligned} u_1(v) &= \left(v - \int_{\underline{b}}^{\bar{b}} b\,dF(b)\right) - c \\ &= v - \frac{\underline{v}}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\ln\left(\frac{2-\tilde{\rho}}{\tilde{\rho}}\right) - c. \end{aligned}$$

$$\begin{aligned} u_2(v) &= \left(v - \int_{\underline{b}}^{\bar{b}} 2b(1-F(b))\,dF(b)\right) - 2c \\ &= v - \frac{\underline{v}\tilde{\rho}^2}{2(1-\tilde{\rho})^2}\left(\frac{2-2\tilde{\rho}}{\tilde{\rho}} + \ln\frac{\tilde{\rho}}{2-\tilde{\rho}}\right) - 2c. \end{aligned}$$

It follows that the cutoffs are:

$$\begin{aligned} c_0(\underline{v}) &= \underline{v} - \frac{\underline{v}}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\ln\left(\frac{2-\tilde{\rho}}{\tilde{\rho}}\right) \\ c_0(\overline{v}) &= \overline{v} - \frac{\underline{v}}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\ln\left(\frac{2-\tilde{\rho}}{\tilde{\rho}}\right) \\ &= c_0(\underline{v}) + (\overline{v} - \underline{v}) > c_0(\underline{v}) \end{aligned}$$

And

$$c_1(\underline{v}) = c_1(\overline{v}) = \underline{v}\frac{\tilde{\rho}}{1-\tilde{\rho}}\left(\frac{1}{2}\frac{1}{1-\tilde{\rho}}\ln\left(\frac{2-\tilde{\rho}}{\tilde{\rho}}\right)-1\right).$$

■

Proof of Proposition 7. The equilibrium is the solution to a system of three equations:

$$\begin{aligned}
\rho(\overline{v}) &= 1-\frac{G(c_1(\underline{v},\tilde{\rho}))}{G(c_0(\overline{v},\tilde{\rho}))}\\
\rho(\underline{v}) &= 1-\frac{G(c_1(\underline{v},\tilde{\rho}))}{G(c_0(\underline{v},\tilde{\rho}))}\\
\tilde{\rho} &= \beta\rho(\overline{v})+(1-\beta)\rho(\underline{v})
\end{aligned}$$

where

$$\beta = \frac{\alpha G(c_0(\overline{v},\tilde{\rho}))}{\alpha G(c_0(\overline{v},\tilde{\rho}))+(1-\alpha)G(c_0(\underline{v},\tilde{\rho}))}.$$

Plugging the first two equations into the third,

$$\tilde{\rho} = \frac{\alpha\left(1-\frac{G(c_1(\underline{v},\tilde{\rho}))}{G(c_0(\overline{v},\tilde{\rho}))}\right)G(c_0(\overline{v},\tilde{\rho}))+(1-\alpha)\left(1-\frac{G(c_1(\underline{v},\tilde{\rho}))}{G(c_0(\underline{v},\tilde{\rho}))}\right)G(c_0(\underline{v},\tilde{\rho}))}{\alpha G(c_0(\overline{v},\tilde{\rho}))+(1-\alpha)G(c_0(\underline{v},\tilde{\rho}))}.$$

With some algebra,

$$\tilde{\rho} = 1-\frac{G(c_1(\underline{v},\tilde{\rho}))}{\alpha G(c_0(\overline{v},\tilde{\rho}))+(1-\alpha)G(c_0(\underline{v},\tilde{\rho}))} \tag{10}$$

One can solve the above equation to find the equilibrium $\tilde{\rho}$ by substituting in it the values of $c_1(\underline{v},\tilde{\rho})$, $c_0(\overline{v},\tilde{\rho})$ and $c_0(\underline{v},\tilde{\rho})$ as a function of $\tilde{\rho}$, which is analogous to the case with price discrimination. The reminder of the proof is similar to that under discrimination. ■

Proof of Lemma 4.

(i) Since $c_0(\overline{v}) > c_0(\underline{v})$ and $c_1(\overline{v}) = c_1(\underline{v})$, it follows that $\rho(\overline{v}) > \rho(\underline{v})$, and since $\tilde{\rho}$ is the weighted average $\rho(\overline{v})$ and $\rho(\underline{v})$, then $\rho(\overline{v}) > \tilde{\rho} > \rho(\underline{v})$. Hence, conditional on search, the buyer with a higher valuation is more likely to have searched only once. (ii) From Lemma 3, this implies that, conditional on search, high valuation consumers pay higher prices, $E[p;\overline{v}] > E[p;\underline{v}]$. In particular, $E[p;\overline{v}] - E[p;\underline{v}] = [\rho^*(\overline{v}) - \rho^*(\underline{v})]\, c_1(\tilde{\rho}^*) > 0$. (iii) Since $c_0(\overline{v}) > c_0(\underline{v})$, the denominator in the right hand side of equation (5 is increasing in α. Hence, the the right hand side of equation (5 is also increasing in α. It follows that the intersection between the right and the left hand side goes up in α, which proofs that $\tilde{\rho}^*$ is increasing in α. ■

Proof of Proposition 8. Assume that the elasticity of the search cost distribution G is everywhere increasing in c. From Proposition 4 we know that $\rho^d(\overline{v}) > \rho^d(\underline{v})$, and from Lemma 4 we know that $\rho^u(\overline{v}) > \tilde{\rho}^u > \rho^u(\underline{v})$. Throughout this proof, we further rely on the fact that, for the case with increasing elasticity, $1-\frac{G(c_1(v,\rho))}{G(c_0(v,\rho))}$ is decreasing in ρ (as guaranteed by the proof of Proposition 3).

Given that for any ρ,

$$1-\frac{G\left(c_1(\underline{v},\rho)\right)}{G\left(c_0(\underline{v},\rho)\right)}<1-\frac{G\left(c_1(\underline{v},\rho)\right)}{\alpha G\left(c_0(\underline{v},\rho)+\overline{v}-\overline{v}\right)+(1-\alpha)\,G\left(c_0(\underline{v},\rho)\right)},$$

it follows that $\tilde{\rho}^u$ has to be to the right of the intersection between $1-\frac{G(c_1(\underline{v},\rho))}{G(c_0(\underline{v},\rho))}$ and ρ. This implies

$$\rho^d\left(\underline{v}\right)=1-\frac{G\left(c_1(\underline{v},\rho^d\left(\underline{v}\right))\right)}{G\left(c_0(\underline{v},\rho^d\left(\underline{v}\right))\right)}<\tilde{\rho}^u.$$

Now, we show $\rho^d\left(\underline{v}\right)>\rho^u\left(\underline{v}\right)$. Since $\rho^d\left(\underline{v}\right)<\tilde{\rho}^u$, and since $1-\frac{G(c_1(\underline{v},\rho))}{G(c_0(\underline{v},\rho))}$ is decreasing in ρ,

$$\rho^d\left(\underline{v}\right)=1-\frac{G\left(c_1(\underline{v},\rho^d\left(\underline{v}\right))\right)}{G\left(c_0(\underline{v},\rho^d\left(\underline{v}\right))\right)}>1-\frac{G\left(c_1(\underline{v},\widetilde{\rho}^u)\right)}{G\left(c_0(\underline{v},\widetilde{\rho}^u)\right)}=\rho^u\left(\underline{v}\right).$$

Last, the result on the participation rates simply follows from the fact that the c_0 threshold is decreasing in ρ, and the raking of the equilibrium ρs reported in Proposition above. The second result follows from combining Lemmas 3 and 4. ■