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Collusion with capacity constraints over the business cycle

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Abstract

This paper investigates the effect of capacity constraints on the sustainability of collusion in markets subject to cyclical demand fluctuations. In the absence of capacity constraints, Haltiwanger and Harrington (1991) [Haltiwanger, J., Harrington, J., 1991. The impact of cyclical demand movements on collusive behavior. Rand Journal of Economics. 22, 89–106.] show that firms find it more difficult to collude during periods of decreasing demand. We find that this prediction can be overturned if firms' capacities are sufficiently small. Capacity constraints imply that punishment profits move procyclically, so that periods of increasing demand may lead to lower losses from cheating even if collusive profits are rising. Haltiwanger and Harrington's main prediction remains valid for sufficiently large capacities.

JEL classification: C73; L13; E32

Keywords: Collusion; Capacity constraints; Business cycles

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1. Introduction

The ability of firms to collude over the business cycle has been a major topic of research in theoretical and empirical industrial organization over the last two decades. The literature has commonly used an infinitely repeated game where firms try to sustain the highest level of profits with credible threats to punish defectors. As firms have a short-run temptation to cheat from the collusive agreement, collusion is stable only if the one-shot deviation gains do not exceed the losses of future collusive profits, net of the value of punishment profits. Changes in demand conditions affect both the gains and losses from cheating, implying that the balance between the two need not remain constant as demand moves over time. Therefore, the state of the business cycle has a crucial effect on the sustainability and profitability of collusive outcomes.

In this paper, we revisit the classical question of whether firms find it more difficult to collude during booms or during recessions. Our point of departure is the model developed by Haltiwanger and Harrington (1991) (hereafter, HH). Holding constant the level of current demand, they show that firms' incentives to deviate are strengthened when future demand is falling, given that the value of the forgone collusive profits is smaller as compared to when demand is rising. Therefore, it is more difficult to sustain collusion during periods of decreasing demand. This result crucially depends on marginal costs being constant in output and symmetric across firms, as this means that punishment profits are zero and therefore invariant to demand movements. If a weakening of demand conditions also leads to a drop in punishment profits, it is no longer clear whether firms would lose less by deviating in periods of falling demand. What this implies for our current purposes is that the effects of future demand movements on the sustainability of collusion are not unambiguous, as they are under the assumption of constant (and symmetric) marginal costs.[1]

By introducing capacity constraints into HH's formulation, we show that the issue of whether firms find it more difficult to collude during booms or recessions is linked to the value of firms' capacities. When capacity constraints are sufficiently tight,[2] firms find it more difficult to collude during booms, whereas the contrary is true when capacities are sufficiently large. Intuitively, when capacity constraints are severe enough, the lack of excess capacity during a boom implies that the future costs of being punished are low. Thus, the losses from cheating decrease even if collusive profits are rising. In contrast, the emergence of excess capacity during a recession makes the punishment threat more severe, and thereby induces an increase in the losses from cheating even if collusive profits decline.

[1] Typically, most of the industries which have been subject to empirical analyses of collusive behavior are characterized either by cyclical cost movements (as the gasoline market analyzed by Borenstein and Shepard (1996)), or by tight capacity constraints (as the aluminum industry analyzed by Rosenbaum (1989), Bresnahan and Suslow (1989), or the cement industry analyzed by Iwand and Rosenbaum (1991) and Rosenbaum and Sukharomana (2001), among others).

[2] All along the paper, we will use the term 'tight' to refer to a level of capacities below a certain threshold but still above the level that would make collusion indistinguishable from one-shot non-cooperative behavior.

1.1. Related papers

This paper also contributes to highlight the importance of the assumptions made in some of the previous papers on collusion. For our current purposes, two assumptions are crucial: first, whether firms are capacity-constrained (or more generally, whether optimal punishment profits depend on demand levels); and second, whether there is some link between current and future demand conditions. The literature on collusion is vast, so we will just review here the papers that are most related to our work.

In a seminal paper, Rotemberg and Saloner (1986) explore optimal collusive pricing assuming that demand is subject to (observable) independent and identically distributed (*i.i.d.*) shocks and that firms' marginal costs are constant in output. Under these assumptions, the current level of demand only affects the sustainability of collusion through its positive effect on firms' short-run temptation to cheat: deviations are more profitable in periods of high demand given that undercutting allows the deviator to capture a larger share of the market. However, the level of current demand has no effect on firms' expectations of future demand, and thus the expected losses from cheating are independent of the level of current demand. Associating a boom (recession) with a period of high (low) demand, Rotemberg and Saloner find that it is more difficult to sustain collusion during booms, when the incentive to deviate is the greatest.[3]

By introducing capacity constraints into Rotemberg and Saloner (1986)'s model, Staiger and Wolak (1992) show that the *price wars during booms relationship* can be reversed. The main reason is that capacity constraints, by limiting the size of the market that a firm can capture by itself, reduce the profitability of defections when demand is sufficiently high. However, by retaining the assumption that the shocks in demand are *i.i.d.*, Staiger and Wolak omit an equally important factor: namely, that the existence of capacity constraints also alters the value of the future losses from cheating through their impact on the severity of future punishments. Brock and Scheinkman (1985) and Compte et al. (2002) highlight the importance of capacity constraints in shaping punishment possibilities. However, since these models assume fixed demand over time, they cannot be used to address the issue of whether booms or recessions are critical for the sustainability of collusion.

Haltiwanger and Harrington (1991) replace the *i.i.d.* assumption by assuming instead that demand is subject to (deterministic) cyclical demand fluctuations.[4] This approach is better suited to understand the influence of the business cycle on firms' pricing behavior since "stronger (weaker) demand tomorrow" is exactly what firms expect if they believe that the economy is in an upturn (downturn). Hence, even if, in the absence of capacity

[3] Given the *i.i.d.* assumption, expected future demand at a period with a high demand realization is lower than current demand. Therefore, in Rotemberg and Saloner's model, a boom (current demand is high) is also a period in which future demand is falling. This should be noted to avoid confusion with our (and HH's) terminology, according to which a boom is a period followed by larger demand levels.

[4] Kandori (1991) assumes correlated demand shocks. More recently, Bagwell and Staiger (1997) assume that the level of market demand alternates stochastically between states of slow (recessions) and fast (boom) growth rates. They show that collusive pricing is weakly procyclical or countercyclical depending on whether market demand growth rates are positively or negatively correlated through time.

constraints, it is still true that the greatest deviation gains are achieved at the peak of the cycle, it is no longer clear whether collusion will be weaker during booms if the greater incentives to deviate are offset by the increasing value of the forgone collusive profits.

Given that our paper is closely related to HH's, it is worthwhile to understand its main insights through the following thought experiment. Consider two points on the cycle with equal demand, but such that demand is increasing in one and decreasing in the other. Clearly, the losses from cheating are greater at the point at which demand is rising, since the near-term profits, which are more heavily weighted, are expected to be higher. Thus, the high cost that would be induced by a price war acts as a deterrent to firms' incentives to cheat. Since such a deterrent is weaker when demand is expected to fall, collusion is more vulnerable during recessions than during booms. However, as already mentioned, the constant marginal cost assumption hides the possibility that future demand movements may also affect future punishment profits, and thus provides an incomplete picture of collusion possibilities in industries where this assumption is not satisfied.

Our model relaxes both the assumption that demand shocks are *i.i.d.* and the assumption that marginal costs are constant in output. By allowing demand to move in cycles (as opposed to Rotemberg and Saloner (1986) and Staiger and Wolak (1992)), we can shed some light on the link between the state of the business cycle and the sustainability of collusion. Furthermore, by introducing capacity constraints, we can provide an answer to the question of whether firms find it more difficult to collude in booms or in recessions for all capacity values, and not only for the limiting case in which capacities tend to infinity (which is equivalent to the assumption of capacity-unconstrained firms, as in HH). By capturing these two elements at a time, our model is able to highlight new results that, although previously conjectured by some authors, have not been so far formalized.[5]

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 provides the analysis and main results, and Section 4 concludes. Proofs are relegated to Appendix A.

2. The model

Consider an industry with n infinitely lived firms, where $n \geq 2$ and finite, which compete in every period $t \geq 1$ by making simultaneous pricing decisions. Firms are symmetric as they offer homogenous products, and face identical cost functions with constant marginal costs (normalized to zero) up to their (exogenously given) symmetric capacity, k. Production above capacity is impossible, i.e. it is infinitely costly. Market demand in period t is represented by the linear demand function $D(p, \theta_t) = \max\{\theta_t - p, 0\}$, where p denotes price and θ_t is a demand parameter.

We further assume that sales are allocated according to the efficient rationing rule.[6] That is, customers buy first from the low-priced firms, until their capacities are exhausted.

[5] See, for instance, Borenstein and Shepard (1996), Cowling (1983), Iwand and Rosenbaum (1991), and Rosenbaum (1989).

[6] This specification has been used, among others, by Kreps and Scheinkman (1983) and Osborne and Pitchik (1986); Davidson and Deneckere (1986) discuss alternative rationing rules.

The aggregate demand faced by the high-priced firms equals market demand at their price less the quantity sold by the lower-priced firms. Ties among the high-priced firms are (symmetrically) broken.

Accordingly, when all firms price at p, total profits are given by

$$\pi^{\mathrm{m}}(p,\theta_t) = p\min\left[D(p,\theta_t), nk\right]. \tag{1}$$

The price that maximizes (1), referred to as the (capacity-constrained) monopoly price, is denoted as $p^{\mathrm{m}}(\theta_t)$. It is easy to check that $p^{\mathrm{m}}(\theta_t)$ and $\pi^{\mathrm{m}}(p^{\mathrm{m}}, \theta_t)$ are increasing in θ_t.

Also, when all firms but one offer prices strictly below p, the profit of the high-priced firm when it charges p is given by

$$\pi^{\mathrm{r}}(p,\theta_t) = p\max\left[0, \min\left[D(p,\theta_t) - (n-1)k, k\right]\right]. \tag{2}$$

Whenever $D(0, \theta_t) > (n-1)\,k$, there exists a unique price which maximizes (2). Such a price is denoted $p^{\mathrm{r}}(\theta_t)$ and it is referred to as the (capacity-constrained) residual monopoly price. If $D(0, \theta_t) \leq (n-1)\,k$, the high-priced firm earns zero profits for all $p \geq 0$. Thus, in this case, we define $p^{\mathrm{r}}(\theta_t)=0$. It is easy to check that $p^{\mathrm{r}}(\theta_t)$ and $\pi^{\mathrm{r}}(p^{\mathrm{r}}, \theta_t)$ are non-decreasing in θ_t and non-increasing in k.

To investigate the impact of demand fluctuations on the sustainability of collusion, we place a similar structure on the intertemporal movement of demand as that of HH. The demand parameter θ_t is assumed to fluctuate in cycles of length $\bar{t}$ according to (3),

$$\theta_t = \begin{cases} \theta_1 & \text{if} \quad t \in \{1, \bar{t}+1, 2\bar{t}+1, \ldots\}, \\ \vdots & \quad\quad \vdots \\ \theta_{\hat{t}} & \text{if} \quad t \in \{\hat{t}, \bar{t}+\hat{t}, 2\bar{t}+\hat{t}, \ldots\}, \\ \vdots & \quad\quad \vdots \\ \theta_{\bar{t}} & \text{if} \quad t \in \{\bar{t}, 2\bar{t}, 3\bar{t}, \ldots\}. \end{cases} \tag{3}$$

We only impose two restrictions on this cycle. First, the demand cycle must be single-peaked. That is, starting at period 1 of the cycle, the demand function is assumed to shift out over time, up to some period $\hat{t}$, and to shift back until it reaches its minimum level at $t=\bar{t}+1$.

A1.

$$\theta_1 < \ldots < \theta_{\hat{t}} \ldots > \theta_{\bar{t}} > \theta_1.$$

Assumption A1 implies that monopoly profits and residual monopoly profits move in the same direction as market demand. That is, monopoly profits increase from period 1 to period $\hat{t}$, and then shift down from period $\hat{t}+1$ to $\bar{t}$. Similarly, residual monopoly profits increase from the first period at which demand at marginal costs exceeds the aggregate capacity of $[n-1]$ firms up to period $\hat{t}$, and then shifts down from period $\hat{t}+1$ until the last point of the cycle at which demand at marginal costs exceeds the aggregate capacity of $[n-1]$ firms. For all other periods, residual monopoly profits equal zero, and are therefore invariant to demand movements.

Last, in order to make the analysis meaningful, aggregate capacity must exceed market demand at the monopoly price at the trough of the cycle. Otherwise, perfect collusion would arise as a one-shot Nash-equilibrium in every period of the cycle.

A2.

$$D(p^{\mathrm{m}},\theta_1)<nk.$$

No further restrictions are imposed on the demand cycle. In particular, it can be symmetric or asymmetric both in the length of the recession and boom, or in the speed at which demand grows during booms or declines during recessions.[7]

Given this demand and cost structure, firms make simultaneous pricing decisions in every period t. With an infinite horizon, a strategy for firm i is an infinite sequence of functions, $\{S_{it}\}_{t=1}^{\infty}$, where S_{it} specifies a cumulative distribution function over $[0,\ \theta_t]$ to be used by firm i in period t as a function of the prices charged by all firms in all previous periods. The payoff function for firm i is the sum of discounted profits, where firms' common discount factor is $\delta\in(0,\ 1)$. All firms are assumed to be risk neutral, and hence aim to maximize their expected payoff. All aspects of the game are assumed to be common knowledge.

3. Analysis and results

The aim of this paper is to highlight the effect of capacity constraints on the sustainability of perfect collusion over the cycle. Therefore, we will first characterize the necessary and sufficient conditions for the path of monopoly prices, $\{p^{\mathrm{m}}(\theta_t)\}_{t=1}^{\infty}$, to be a subgame perfect equilibrium outcome of the infinitely repeated game described above.

We consider the trigger strategy that prescribes firms to price at the monopoly price[8] in every period as long as no firm has deviated in previous periods, and to revert forever to the static Nash equilibrium strategy in the event of a deviation. At the static Nash equilibrium, each firm receives its minmax profits (Brock and Scheinkman, 1985; Kreps and Scheinkman, 1983), i.e. the lowest value of profits a firm can be credibly driven down to.[9] Since more severe punishment threats would not be credible, Nash reversion constitutes an optimal penal code.

[7] Implicit in this formulation is the assumption that demand movements are not so strong so as to induce exit or entry, nor capacity expansions or contractions. Endogenizing market structure and capacity levels is out of the scope of the paper. See Rotemberg and Woodford (1992) for a general equilibrium approach and Staiger and Wolak (1992) for analysis of collusion with endogenous capacities.

[8] More precisely, the strategy calls firms to play a degenerate mixed strategy with a masspoint of size one at the monopoly price.

[9] In detail, if $\theta_t\leq(n-1)\,k$, the static Nash equilibrium has all firms pricing at (zero) marginal costs and earning zero profits; if $(n+1)\,k\leq\theta_t\leq 2nk$, the static Nash equilibrium has all firms pricing at the market clearing price (θ_t-nk); thereby making profits $(\theta_t-nk)k$. Last, if $(n-1)k\leq\theta_t\leq(n+1)k$ the static Nash equilibrium involves mixed-strategy pricing with expected profits given by $\left[\frac{\theta_t-(n-1)k}{2}\right]^2$.

Accordingly, the path of monopoly prices is a subgame perfect equilibrium outcome if and only if the following condition is satisfied,

$$L^{\mathrm{m}}(t;\delta) \geq G^{\mathrm{m}}(t) \forall t, \tag{4}$$

where $L^{\mathrm{m}}(t;\ \delta)$ denotes the present discounted value of the losses from cheating from period $t+1$ onwards and $G^{\mathrm{m}}(t)$ represents the one-shot deviation gain in period t.

Formally,

$$L^{\mathrm{m}}(t;\delta) = \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} L^{\mathrm{m}}(\tau);$$

$$L^{\mathrm{m}}(\tau) = \frac{1}{n}\pi^{\mathrm{m}}(p^{\mathrm{m}},\theta_\tau) - \pi^{\mathrm{r}}(p^{\mathrm{r}},\theta_\tau). \tag{5}$$

and

$$G^{\mathrm{m}}(t) = p^{\mathrm{m}}(\theta_t)\min\left\{D(p^{\mathrm{m}},\theta_t),k\right\} - \frac{1}{n}\pi^{\mathrm{m}}(p^{\mathrm{m}},\theta_t). \tag{6}$$

Eq. (5) states that the losses from cheating in period t are given by the present discounted value of the difference between each firm's (symmetric) share of monopoly profits, and the static Nash equilibrium profits in all the periods $\tau > t$. It is easy to check that the profits that a firm makes at the static Nash equilibrium are the same as if all its rivals priced at zero and the firm maximized its profits over the residual demand (Brock and Scheinkman, 1985), so that $\pi^{\mathrm{r}}(p^{\mathrm{r}},\ \theta_\tau)$ represents the static Nash equilibrium profits in period $\tau > t$. Henceforth, we will refer to $L^{\mathrm{m}}(\tau)$ as the one-shot losses from cheating in period $\tau > t$. Last, as implicit in Eq. (6), the optimal deviation in period t is to slightly undercut $p^{\mathrm{m}}(\theta_t)$, and this results in profits $p^{\mathrm{m}}(\theta_t)\min\{D(p^{\mathrm{m}},\ \theta_t),\ k\}$ rather than $(1/n)\pi^{\mathrm{m}}(p^{\mathrm{m}},\ \theta_t)$.

The incentive compatibility constraint, (4), can be solved in terms of the discount factor, δ. As is already standard, the path of monopoly prices is subgame perfect if and only if the discount factor is sufficiently large.

Proposition 1. *There exists $\hat{\delta} \in (0,\ 1)$ such that the price path $\{p^m(\theta_t)\}_{t=1}^{\infty}$ is supportable by subgame perfect equilibria if and only if $\delta\ \in (\hat{\delta},\ 1)$.*

Proposition 1 is used to implicitly define the period of the cycle when firms find it more difficult to sustain perfect collusion. When the discount factor exceeds $\hat{\delta}$, the incentive compatibility constraint (4) is satisfied with a strict inequality in all periods. When it equals $\hat{\delta}$, there exists some point(s) of the cycle, which we will denote t^*, at which the incentive compatibility constraint (4) is satisfied with a strict equality. Therefore, as the discount factor is slightly reduced below $\hat{\delta}$, t^* is the first period(s) at which the monopoly price cannot not be sustained. We thus refer to any such t^* as *the critical point of the cycle*.

In order to assess whether t^* belongs to the boom or to the recession, we first need to investigate how the value of the one-shot losses and gains from cheating depend on the level of firms' capacities.

For this purpose, let us first assess how the one-shot losses from cheating $L^{\mathrm{m}}\ (t)$ vary as a function of the demand parameter θ_t. Consider a situation in which capacities are so large relative to demand that punishment profits are driven down to zero. In this case, the one-shot losses from cheating are just equal to the value of the forgone monopoly profits, which are clearly increasing in demand. However, for smaller capacities firms are capacity-constrained and unable to drive punishment profits to zero. Hence, a strengthening of demand conditions not only leads to an increase in monopoly profits, but also to less severe punishments. This implies that the increase in the one-shot losses from cheating is at least partially offset. If capacity constraints are sufficiently tight, the increase in monopoly profits is more than offset by the increase in punishment profits, so that the losses from cheating start to decrease as demand conditions strengthen.

We can perform the same analysis to understand the impact of demand fluctuations on the value of the one-shot deviation gains, $G^{\mathrm{m}}(t)$. Consider first a situation in which each firm's capacity is large enough so that a defector would have enough capacity to serve all demand at the monopoly price. Thus, the larger demand, the higher the monopoly profits, and the higher the one-shot deviation gains. For smaller capacity values, the deviator would be capacity-constrained to expand its production up to the monopoly quantity. Since, as a function of demand, the increase in the deviator's profits is of lower-order magnitude than the increase in monopoly profits, the rate of growth of the one-shot deviation gains starts to decrease. If capacities are small enough, the former effect dominates the latter, and implies that the one-shot deviation gains are decreasing in demand.

The following Lemma identifies the critical capacity value above (below) which the one-shot losses and gains from cheating in a given period are increasing (decreasing) in the demand parameter θ_t.

Lemma 1. *The one-shot losses and gains from cheating, $L^m(t)$ and $G^m(t)$, are increasing in θ_t if $k \geq \theta_t/n$, and decreasing otherwise.*

Building on these insights, we can now assess whether the critical point of the cycle for perfect collusion belongs to the boom or to the recession, and how this depends on the value of firms' capacities.

Proposition 2. *If $k \leq \theta_1/n$, the critical point of the cycle belongs to the boom, $t^* \in \{1,\ldots,\hat{t}-1\}$; and if $k \geq \theta_{\hat{t}}/n$, it belongs to the recession, $t^* \in \{\hat{t},\ldots,\bar{t}\}$.*

Proposition 2 shows that the critical period of the cycle belongs to the boom when each firm's capacity is small enough, and it belongs to the recession when each firm's capacity is large enough.

As shown in Lemma 1, when capacities are large enough, i.e. $k \geq \theta_{\hat{t}}/n$, both the losses from cheating and the one-shot deviation gains are increasing in θ_t in all periods of the cycle. Hence, the same logic as in HH applies. Namely, for any point at which demand is rising, t^{B}, one can always find a point at which demand is falling, t^{R}, that yields at least as high a one-shot gain from defection. The losses from cheating would be greater at t^{B}, since the near term losses, which are more heavily weighted, exceed those at t^{R}. Therefore, as δ is slightly reduced below $\hat{\delta}$, the first point at which the monopoly price cannot be sustained belongs to the recession.

On the other hand, just the opposite occurs when capacities are small enough, i.e. when $k \leq \theta_1/n$. In this case, as shown in Lemma 1, the one-shot losses and gains from cheating are decreasing in θ_t in all periods of the cycle. Therefore, for any point at which demand is falling, t^{R}, one can always find a point at which demand is rising, t^{B}, that yields at least as high one-shot gain from defection. Now, the losses from cheating would be greater at t^{R}, since the near term losses, which are more heavily weighted, exceed those at t^{B}. Therefore, as δ is slightly reduced below $\hat{\delta}$, the first point at which the monopoly price cannot be sustained belongs to the boom, and not to the recession.

When capacities lay in the interval $(\theta_1/n, \theta_{\hat{t}}/n)$, there may be some periods at which the losses and gains from cheating are decreasing in demand, and others in which they are increasing in demand. Hence, we cannot apply the same reasoning as above. Intuition suggests that there should be monotonicity between the level of firms' capacities and the location of the critical point for perfect collusion. The numerical exercise presented below confirms that there are examples in which this monotonicity holds.

We have set $n=4$; and have assumed that the demand parameter θ_t moves in a symmetric eight-period cycle. Specifically, the demand parameter takes the following values, $\theta_t = \{160, 170, 180, 190, 200, 190, 180, 170\}$. The peak occurs at period $t=5$, periods $t \in \{1, 2, 3, 4\}$ belong to the boom, and periods $t \in \{5, 6, 7, 8\}$ belong to the recession. We have considered variations over k to compute, for each capacity level, the critical period for perfect collusion t^* (upper panel in Fig. 1) and the critical discount factor for perfect collusion $\hat{\delta}$ (lower panel in Fig. 1).

Confirming our theoretical findings, the critical period for perfect collusion belongs to the boom for all $k \leq \theta_1/n = 40$, and to the recession for all $k \geq \theta_{\hat{t}}/n = 50$. Furthermore, there

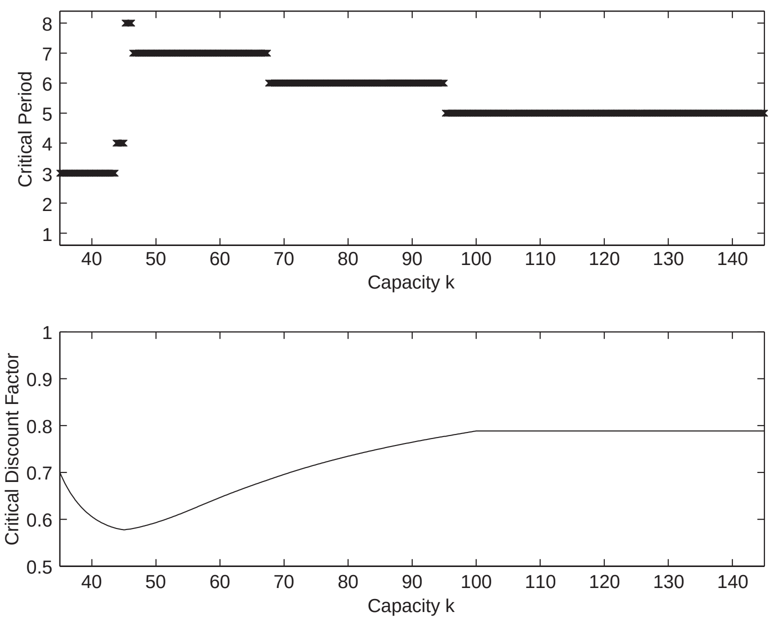

Fig. 1. The critical period, t^*, and critical discount factor for perfect collusion, $\hat{\delta}$, as a function of k.

exists a unique capacity level, $k=45\in(\theta_1/n,\ \theta_{\hat{i}}/n)$ below (above) which the boom (recession) is critical.

Last, consistently with Brock and Scheinkman (1985), Fig. 1 depicts a non-monotonic relationship between the critical discount factor and the value of firms' capacities, whose value first decreases as capacity grows up to $k=45$ (when the recession becomes critical), and then increases up to the point in which the defector is no longer capacity-constrained to serve all the market at the monopoly price, $k=100$.

4. Conclusions

The main objective of this paper has been to identify whether firms find it more difficult to collude during booms or during recessions, and to assess how this depends on the level of firms' capacities. By introducing capacity constraints in a model similar to Haltiwanger and Harrington (1991), we have shown that firms find it more difficult to collude in booms for small capacities and recessions for large capacities. The reason underlying this result is as follows: when firms face severe capacity constraints, the impact of demand fluctuations on the value of future punishment profits is greater than its effect on the value of the forgone collusive profits; hence, periods of expanding demand give rise to lower losses from cheating, which make collusion more difficult during booms rather than recessions. When capacity constraints are not severe enough, the increase in the value of future punishment profits during a boom is not sufficient to outweigh the faster increase in firms' collusive profits. Thus, firms find it more difficult to collude during recessions even if capacity constraints play a role in reducing the one-shot deviation gains and the severity of optimal punishments.

The main implication of this analysis for empirical work is that the signs of the effects of future demand on current prices are not unambiguous, as these also depend on the value of firms' capacities. This suggests that the projected link between the level of future demand and the value of firms' capacities could be used as an additional determinant of the intertemporal price path in collusive industries subject to cyclical demand fluctuations.

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Appendix A. Proofs

Proof of Proposition 1. The subgame perfect equilibrium conditions (as expressed in (4)) take the following form:

$$L^{\mathrm{m}}(t;\delta)=\frac{1}{1-\delta^{\bar{t}}}\left[\delta L^{\mathrm{m}}(t+1)+\ldots+\delta^{\bar{t}-t}L^{\mathrm{m}}(\bar{t})+\ldots+\delta^{\bar{t}}L^{\mathrm{m}}(t)\right]\geq G^{\mathrm{m}}(t).$$

First notice that $L^{\mathrm{m}}(t;\,0)=0\leq G^{\mathrm{m}}(t)$, with a strict inequality at least in period 1 of the cycle. Also, $\lim_{\delta\to 1}L^{\mathrm{m}}(t;\,\delta)=\infty>G^{\mathrm{m}}(t)$ and $(\partial L^{\mathrm{m}}(t;\,\delta)/\partial\delta)>0=(\partial G^{\mathrm{m}}(t)/\partial\delta)$. By the continuity of $L^{\mathrm{m}}(t;\,\delta)$ in δ, there exists $\hat{\delta}(t)\in(0;\,1)$ such that $L^{\mathrm{m}}(t;\,\delta)\geq G^{\mathrm{m}}(t)$ if and only if $\delta\geq\hat{\delta}(t)$. Hence, the price path of monopoly prices is a subgame perfect equilibrium outcome if and only if $\delta\geq\hat{\delta}=\max\{\hat{\delta}(1),\ldots,\hat{\delta}(\bar{t})\}$. Since $\hat{\delta}(t)\in(0,\,1)\forall t$, then $\hat{\delta}\in(0,\,1)$. □

Proof of Lemma 1. The losses and gains from cheating are given by:

$$L^{\mathrm{m}}(t)=\begin{cases}\left[\frac{\theta_t}{2}\right]^2\frac{1}{n} & \text{if } \theta_t<(n-1)k\\ \left[\frac{\theta_t}{2}\right]^2\frac{1}{n}-\left[\frac{\theta_t-(n-1)k}{2}\right]^2 & \text{if } (n-1)k<\theta_t<(n+1)k\\ \left[\frac{\theta_t}{2}\right]^2\frac{1}{n}-[\theta_t-kn]k & \text{if } (n+1)k<\theta_t<2nk\end{cases}$$

$$G^{\mathrm{m}}(t)=\begin{cases}\left[\frac{\theta_t}{2}\right]^2\frac{n-1}{n} & \text{if } \theta_t<2k\\ \frac{\theta_t}{2}k-\left[\frac{\theta_t}{2}\right]^2\frac{1}{n} & \text{if } 2k<\theta_t<2nk\end{cases}.$$

Differentiating the above expressions with respect to θ_t;

$$\frac{\partial L^{\mathrm{m}}(t)}{\partial\theta_t}=\begin{cases}\frac{\theta_t}{2n} & \text{if } \theta_t<(n-1)k\\ \frac{1}{2}[n-1]\left[k-\frac{\theta_t}{n}\right] & \text{if } (n-1)k<\theta_t<(n+1)k\\ \frac{\theta_t}{2n}-k & \text{if } (n+1)k<\theta_t<2nk\end{cases}$$

$$\frac{\partial G^{\mathrm{m}}(t)}{\partial\theta_t}=\begin{cases}\theta_t\frac{n-1}{2n} & \text{if } \theta_t<2k\\ \frac{1}{2}\left[k-\frac{\theta_t}{n}\right] & \text{if } 2k<\theta_t<2nk\end{cases}.$$

Hence, $L^{\mathrm{m}}(t)$ and $G^{\mathrm{m}}(t)$ are increasing in θ_t if and only if $k\geq\theta_t/n$. □

Proof of Proposition 2. We follow HH's proof of Theorem 5, and introduce several changes where needed. Let t^* be defined by $\hat{\delta}=\hat{\delta}(t^*)$ (where $\hat{\delta}(t)$ has been defined in the Proof of Proposition 1). To prove the Proposition, we then need to show that if $k\leq\theta_1/n$, $\hat{\delta}>\hat{\delta}(t)$ for all $t\in\{\hat{t},\ldots,\bar{t}\}$, and if $k\geq\theta_{\hat{t}}/n$, $\delta>\delta(t)$ for all $t\in\{1,\ldots,\hat{t}-1\}$. Since t^* exists, then it must lie in $\{1,\ldots,\hat{t}-1\}$ if $k\leq\theta_1/n$, and in $\{\hat{t},\ldots,\bar{t}\}$ if $k\geq\theta_{\hat{t}}/n$.

Define $f(t)$ as follows:

$$f(t)\equiv\max\{\tau\,|\,\theta_\tau\geq\theta_t,\tau\in\{t+1,\ldots,\bar{t}\}\},t\in\{1,\ldots,\hat{t}-1\}.$$

$f(t)$ is the latest point of the cycle at which the demand parameter is at least as great as the demand parameter at a period t belonging to the boom. Given that the single peak of the cycle is attained at $\hat{t}$, it is clear that $f(t)$ belongs to the recession.

The method of proof will be to show that the difference

$$\left[L^{\mathrm{m}}(t;\delta)-G^{\mathrm{m}}(t)\right]-\left[L^{\mathrm{m}}(f(t);\delta)-G^{\mathrm{m}}(f(t))\right], \tag{7}$$

is negative if $k\leq\theta_1/n$, and positive if $k\geq\theta_{\hat{t}}/n$.

We prove the result for the case $k\leq\theta_1/n$. The proof for the second case is similar, with only a change in the sign of the inequalities.

Let $t^{\mathrm{B}}\in\{1,\ldots,\hat{t}-1\}$ and $t^{\mathrm{R}}=f(t^{\mathrm{B}})$. Then,

$$\left[L^{\mathrm{m}}\left(t^{\mathrm{B}};\delta\right)-G^{\mathrm{m}}\left(t^{\mathrm{B}}\right)\right]=\frac{1}{1-\delta^{\bar{t}}}\left[\delta L^{\mathrm{m}}\left(t^{\mathrm{B}}+1\right)+\ldots+\delta^{\bar{t}}L^{\mathrm{m}}\left(t^{\mathrm{B}}\right)\right]-G^{\mathrm{m}}\left(t^{\mathrm{B}}\right), \tag{8}$$

$$\left[L^{\mathrm{m}}\left(t^{\mathrm{R}};\delta\right)-G^{\mathrm{m}}\left(t^{\mathrm{R}}\right)\right]=\frac{1}{1-\delta^{\bar{t}}}\left[\delta L^{\mathrm{m}}\left(t^{\mathrm{R}}+1\right)+\ldots+\delta^{\bar{t}}L^{\mathrm{m}}\left(t^{\mathrm{R}}\right)\right]-G^{\mathrm{m}}\left(t^{\mathrm{R}}\right). \tag{9}$$

By the definition of $f(t)$, we know by Lemma 1, that if $k\leq\theta_1/n$, then $G^{\mathrm{m}}(t^{\mathrm{B}})>G^{\mathrm{m}}(t^{\mathrm{R}})$. Hence, the difference between (8) and (9) is negative if

$$\delta L^{\mathrm{m}}\left(t^{\mathrm{B}}+1\right)+\ldots+\delta^{\bar{t}}L^{\mathrm{m}}\left(t^{\mathrm{B}}\right)<\delta L^{\mathrm{m}}\left(t^{\mathrm{R}}+1\right)+\ldots+\delta^{\bar{t}}L^{\mathrm{m}}\left(t^{\mathrm{R}}\right). \tag{10}$$

Define:

$$A\equiv\delta L^{\mathrm{m}}\left(t^{\mathrm{B}}+1\right)+\ldots+\delta^{t^{\mathrm{R}}-t^{\mathrm{B}}}L^{\mathrm{m}}\left(t^{\mathrm{R}}\right), \tag{11}$$

$$B\equiv\delta L^{\mathrm{m}}\left(t^{\mathrm{R}}+1\right)+\ldots+\delta^{\bar{t}-t^{\mathrm{R}}+t^{\mathrm{B}}}L^{\mathrm{m}}\left(t^{\mathrm{B}}\right). \tag{12}$$

By these definitions, condition (10) is equivalent to

$$A+\delta^{t^{\mathrm{R}}-t^{\mathrm{B}}}B<B+\delta^{\bar{t}-t^{\mathrm{R}}+t^{\mathrm{B}}}A.$$

Rearranging terms,

$$\frac{A}{1-\delta^{t^{\mathrm{R}}-t^{\mathrm{B}}}}<\frac{\mathrm{B}}{1-\delta^{\bar{t}-t^{\mathrm{R}}+t^{\mathrm{B}}}}. \tag{13}$$

The expression on the left hand side of (13) is the present discounted value of the stream of the losses from cheating $\{L^{\mathrm{m}}(t^{\mathrm{B}}+1),\ldots,L^{\mathrm{m}}(t^{\mathrm{R}})\}$ made every $t^{\mathrm{R}}-t^{\mathrm{B}}$ periods, and the right hand side is the present discounted value of the stream of the losses from cheating $\{L^{\mathrm{m}}(t^{\mathrm{R}}+1),\ldots,L^{\mathrm{m}}(t^{\mathrm{B}})\}$ made every $\bar{t}-t^{\mathrm{R}}+t^{\mathrm{B}}$ periods. Since $t^{\mathrm{R}}=f(t^{\mathrm{B}})$ and $k\leq\theta_1/n$, by Lemma 1 it is then true that $L^{\mathrm{m}}(t')<L^{\mathrm{m}}(t'')\,\forall t'\in\{t^{\mathrm{B}}+1,\ldots,t^{\mathrm{R}}\}$, $\forall t''\in\{t^{\mathrm{R}}+1,\ldots,t^{\mathrm{B}}\}$, which implies that $L^{\mathrm{m}}(t^{\mathrm{B}},\ \delta)<L^{\mathrm{m}}(t^{\mathrm{R}},\ \delta)$. This proves that if $k\leq\theta_1/n$, then $L^{\mathrm{m}}(t;\delta)-G^{\mathrm{m}}(t)<L^{\mathrm{m}}(f(t);\ \delta)-G^{\mathrm{m}}(f(t))\,\forall t\in\{1,\ldots,\hat{t}-1\}$. □

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